Adaptive Optics Lectures

1. Atmospheric turbulence

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Resources

CTIO:

www.ctio.noao.edu/~atokovin/tutorial/index.html

CFHT AO tutorial:

http://www.cfht.hawaii.edu/Instruments/Imaging/AOB/other-aosystems.html

Wikipedia:

https://en.wikipedia.org/wiki/Adaptive_optics

Plan

- Physics of OPTICAL turbulence
- Local parameters & Kolmogorov law
- Wavefront statistics
- Imaging as interference: PSF, OTF
- Imaging through turbulence
- Tip, tilt and beyond (Zernike modes)

Turbulence

- Hydrodynamics: unstable flow breaks up into eddies. The kinetic energy is transferred from large to small scales in a cascade, dissipates eventually by viscosity.
- “Dynamical” turbulence has no optical effect. Fluctuations of the air refractive index are caused primarily by the temperature differences. Turbulence defines the statistics of ΔT.

$\Delta T \rightarrow$ refr.index $\rightarrow$ wavefront $\rightarrow$ image

Kolmogorov's law

$$<\Delta n(r)^2> = C_n^{\,2}\, r^{2/3}$$

This is the *definition* of $C_n^{\,2}$ and $C_T^{\,2}$.

$$<\Delta T(r)^2> = C_T^{\,2}\, r^{2/3}$$

Local turbulence strength

Typical values: 10^{-16} $m^{-2/3}$ and 10^{-6} K^2 $m^{-2/3}$ (1 mK/m)

$$C_n^{\,2} = (80.10^{-6}\, P/T^2)^2\, C_T^{\,2}$$

$\Delta n \sim 7.8\ 10^{-7} \Delta T$

n~1.0003 at sea level

P:millibars, T: K

Limits of the Kolmogorov law

- Saturates at large scale $r>L_0$, L_0~10m (outer scale).

Otherwise infinite fluctuations!

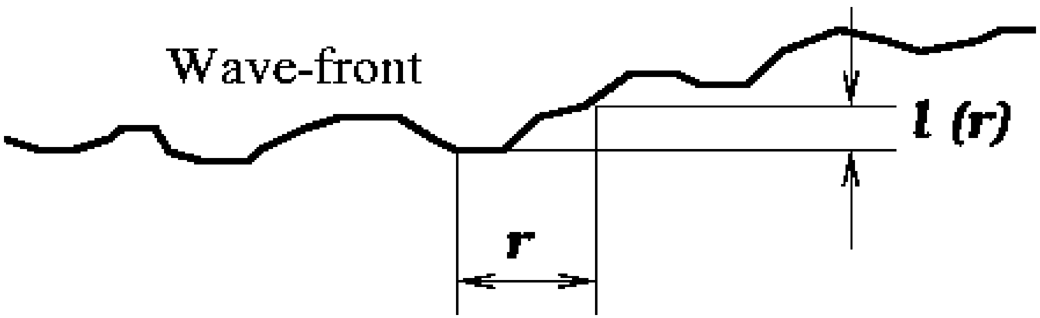
- Break at small scale l_0 (inner scale <1cm)
- Implies random stationary process, in fact turbulence is not stationary
- Does not work if no energy cascade (in the dome, gravity waves, etc.)

Propagation through turbulence

- Fresnel diffraction: $r_{Fresnel} = \sqrt{\lambda z}$ (~10cm for 500nm @ 10km)
- At $r >> r_{Fresnel}$: geometric optics (sum phase lags on the path)
- At $r < r_{Fresnel}$: diffraction, intensity fluctuations (scintillation)

$$\Delta l = \int \Delta n(z)\, dz$$

$$\Delta\phi = \Delta l * (2\pi/\lambda)$$

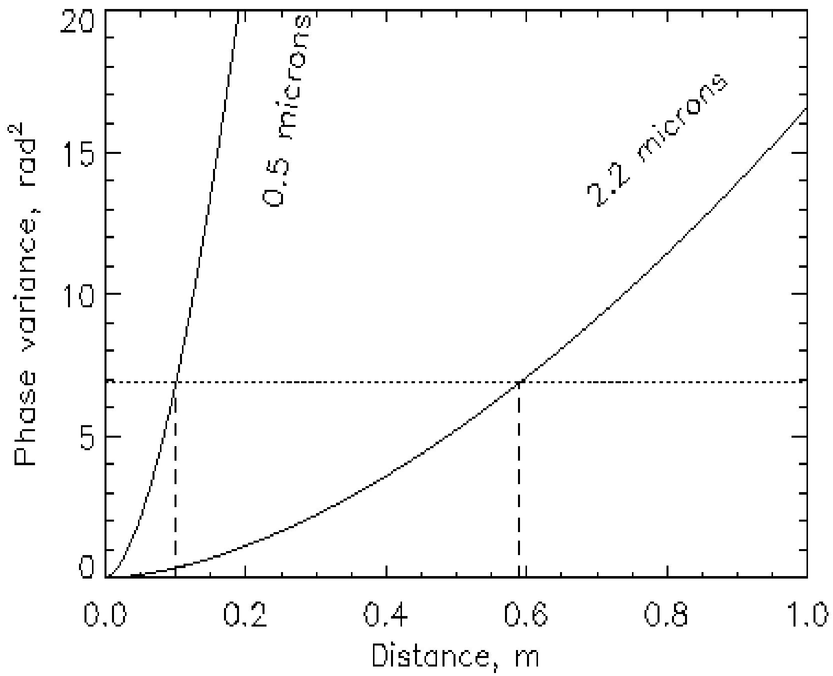


Phase structure function

$$D_\phi(r) = \langle[\Delta\phi(x+r) - \Delta\phi(x)]^2\rangle = 6.88(r/r_0)^{5/3}$$

$r_{Fresnel} < r < L_0$

Fried parameter (coherence length) r_0



$$r_0^{-5/3} = 0.423(2\pi/\lambda)^2 J$$

$$J = \int C_n^2(z)\, dz$$

J = turbulence integral

Point Spread Function (PSF) & Optical Transfer Function (OTF)

PSF $P(\alpha)$ = image of the point source (real)

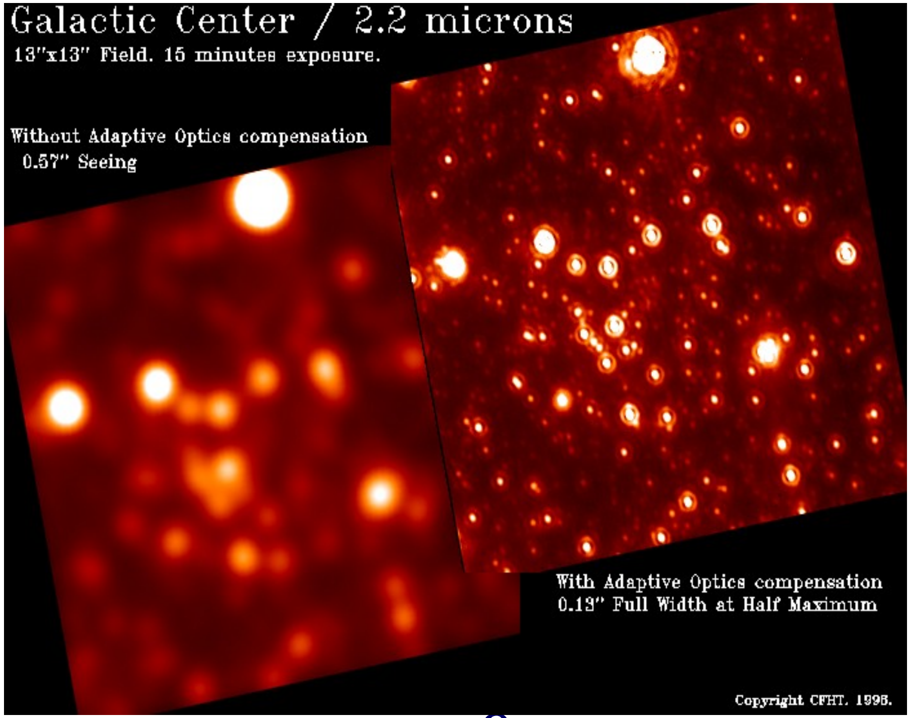
OTF = Fourier Transform (PSF), $\tilde{O}(f)$ complex

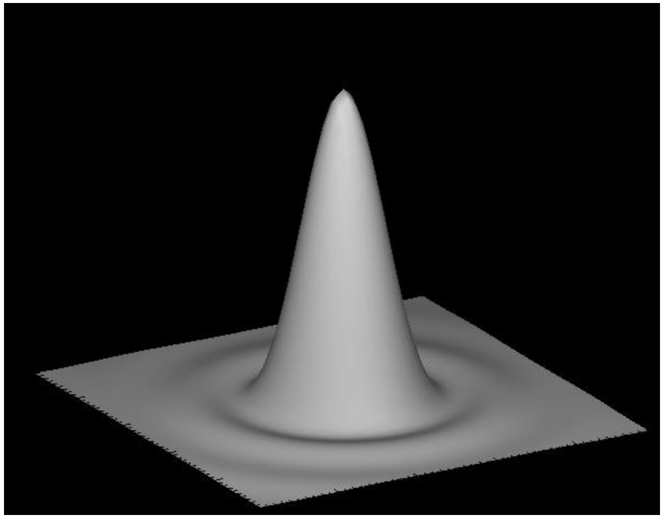
$$I(\alpha) = \int O(\beta)\, P(\alpha-\beta)\, d\beta$$

$$\tilde{I}(f) = \tilde{O}(f) \,.\, \tilde{P}(f)$$

Convolution in image space

Product in Fourier space

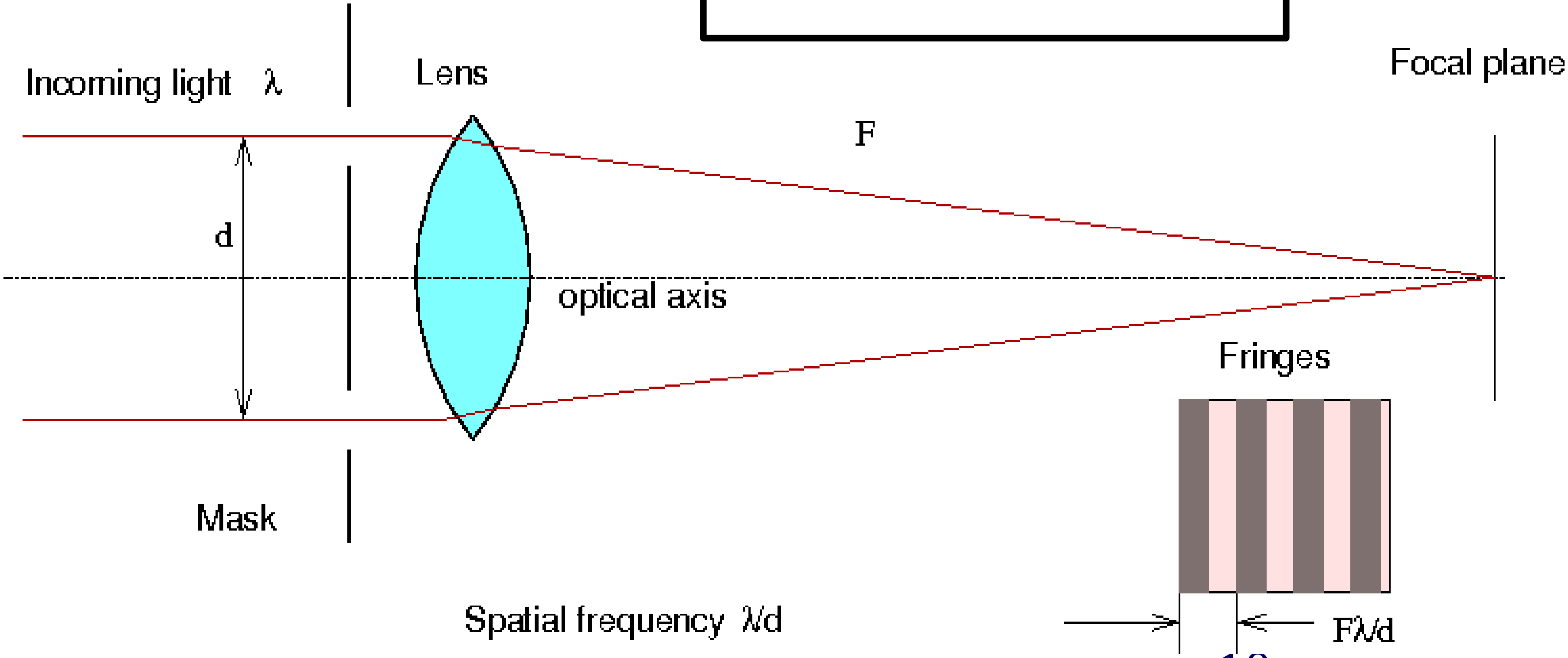




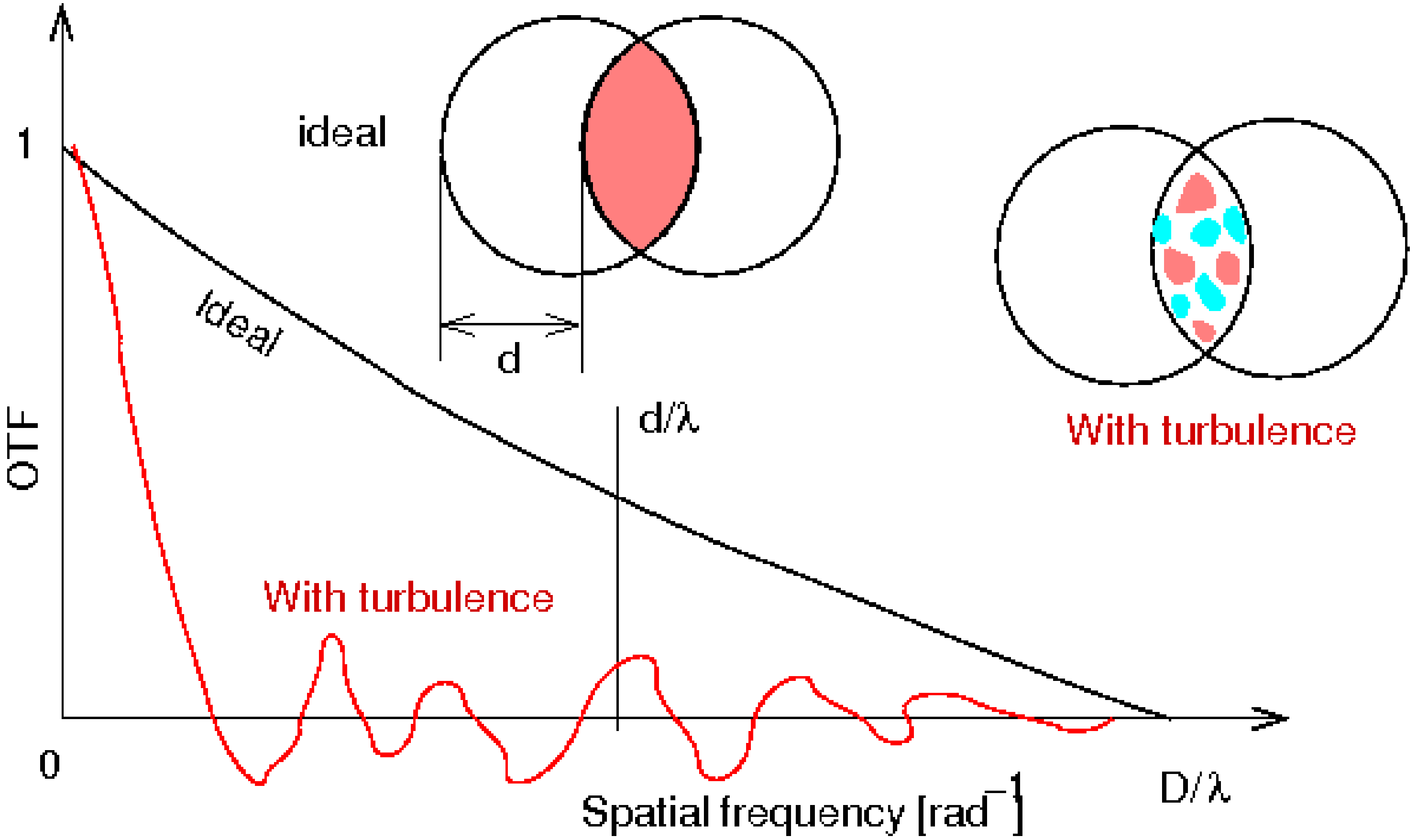
Imaging as interference

$p = \lambda/d$ [radian]

$f = d/\lambda$ [radian^{-1}]



OTF with and without turbulence



Atmospheric OTF

At long exposures, the OTF and PSF are averaged.

1. at $d << r_0$, phase fluctuations $<< \lambda$, coherent
2. at $d >> r_0$, phase fluctuations $> \lambda$, incoherent

Coherence = $\exp[- \langle \Delta\phi^2 \rangle / 2]$

$$P^{\sim}(f) = \exp[- D_\phi(\lambda f)/2] = \exp[-3.44\ (\lambda f / r_0)^{5/3}]$$

This is atmospheric OTF for long exposures.

“Seeing”

Atmospheric PSF has FWHM $\varepsilon = 0.98\lambda/r_0$ [rad]

The PSF is not Gaussian, but similar

“Seeing” β is the FWHM at 500nm at zenith:

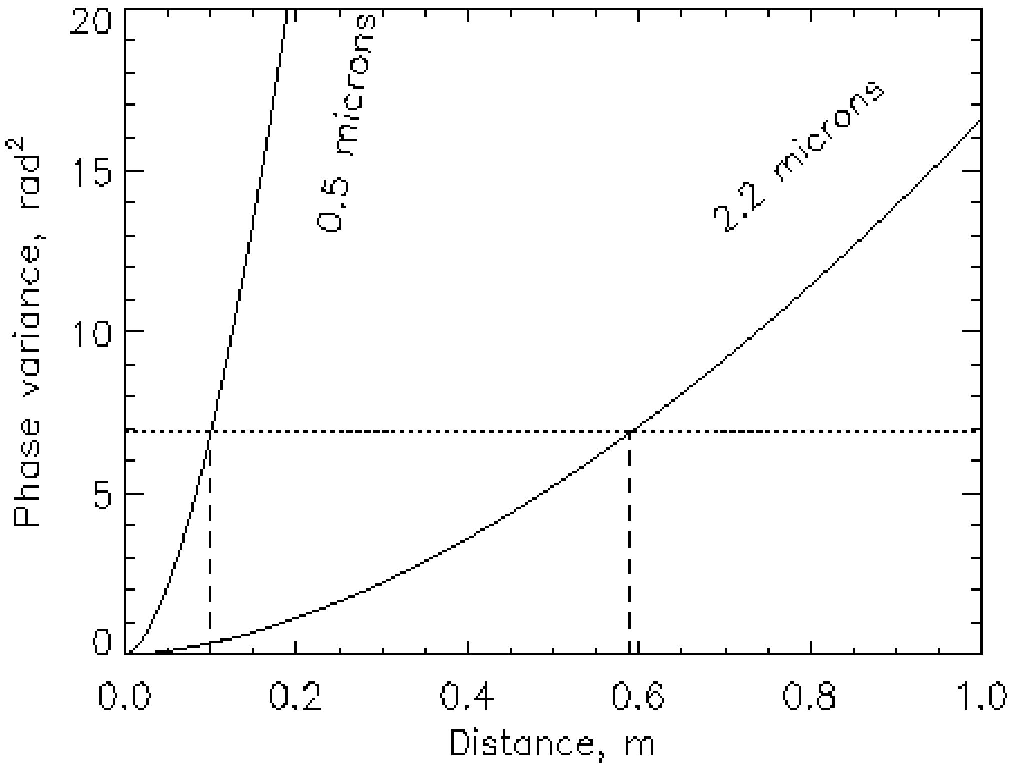
$$\beta = 0.101/r_0[\text{m}] = (J/6.8\ 10^{-13})^{3/5}\ [\text{arcsec}]$$

Dependence on zenith angle: $\beta \sim (\sec z)^{3/5}$

Dependence on wavelength: $\beta \sim \lambda^{-1/5}$

Numerical example

Seeing 1” → r_0 (0.5μm)=10.1cm, J=6.8 10^{-13} $m^{1/3}$.



r_0(2.2 μm)=0.60m

ε (2.2 μm)= 0.74”

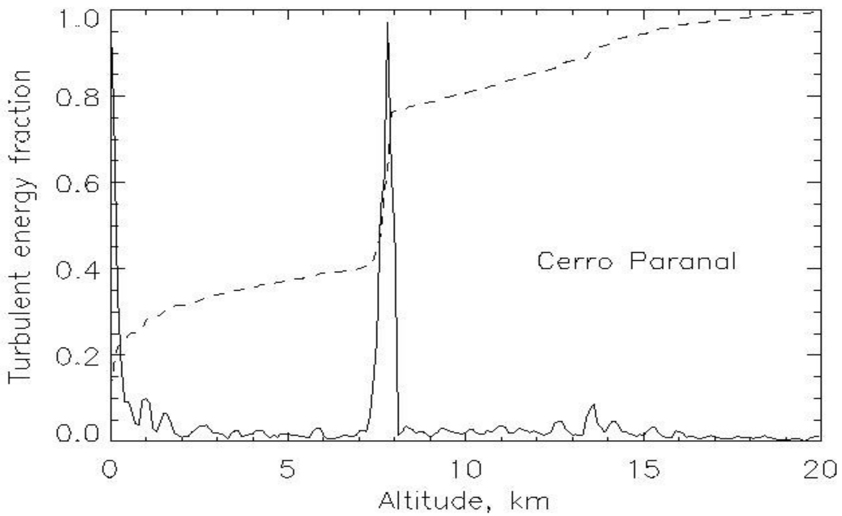
Finite outer scale improves resolution (~10% in the visible, ~2x in the IR)

Turbulence profile

$C_n^2(h)$ and wind speed V(h) describe the turbulence:

1. Surface layer (h<100m, includes dome)
2. Boundary layer (h<1km)
3. Free atmosphere (1km < h <20km)

Optical turbulence is produced by mixing air with different temperatures. “Layers” arise in wind-shear zones.



Isoplanatic angle

Wavefronts from two stars are similar if $\theta < \theta_0$

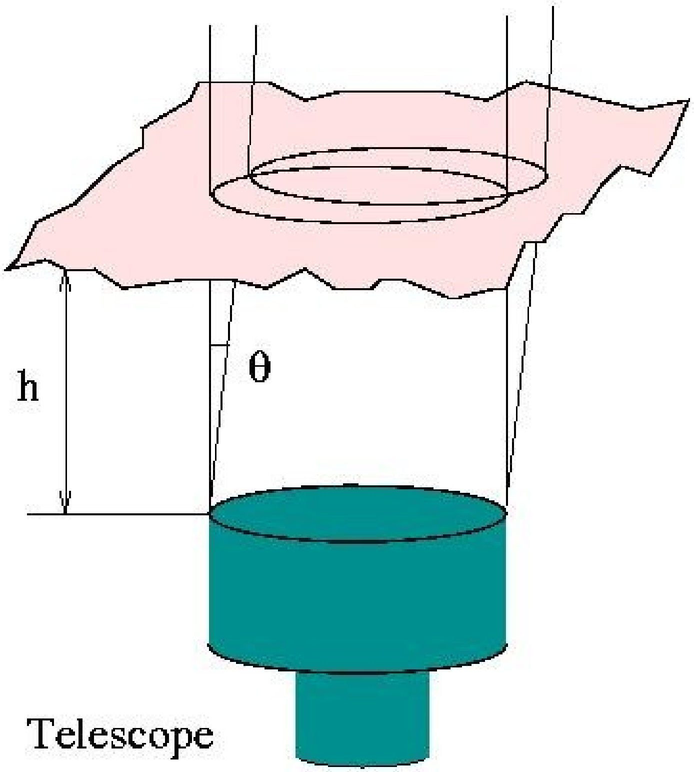
$$\theta_0 = 0.31\, r_0/H$$

H = average height

$H = 5km$, $r_0 = 0.1m$

$\rightarrow \theta_0 = 1.2''$

Also: cone effect with LGS!



Atmospheric time constant

Each turbulent layer is “dragged” by the wind.

$$\tau_0 = 0.31\, r_0/V$$

V = average wind speed

V=20m/s, r_0=0.1m

$\rightarrow\ \tau_0 =\ 1.6$ms

When one layer dominates, the wavefront “moves”, otherwise it “boils”.

Zernike aberrations

Wavefront on a circular pupil can be represented by the sum of basis functions. Zernike basis is the most popular.

Num.	Aberration
1	Piston Z=1
2	Tip $Z=2\rho \cos \theta$
3	Tilt $Z=2\rho \sin \theta$
4	Defocus $Z=\sqrt{3}(2\rho^2-1)$
5	45-astig. $Z= \sqrt{6}\, \rho^2 \sin(2\theta)$

$$\phi(\vec{r}) = \sum_{j=1}^{\infty} a_j \; Z_j(\vec{r}),$$

The Zernike polynomials are defined in polar coord. $Z(\rho,\theta)$ ($0 < \rho < 1$).
Variance=1 (Noll), Cov=0 (ortho-normal basis)

Atmospheric Zernike modes

Zernike coefficients are random variables. The Kolmogorov turbulence model relates variances and covariances to the seeing.

$$\langle a_i a_j \rangle = c_{ij} \left(\frac{D}{r_0} \right)^{5/3},$$

Noll's coefficients $c_{i,j}$: 0.449 for tip & tilt, 0.0232 for defocus

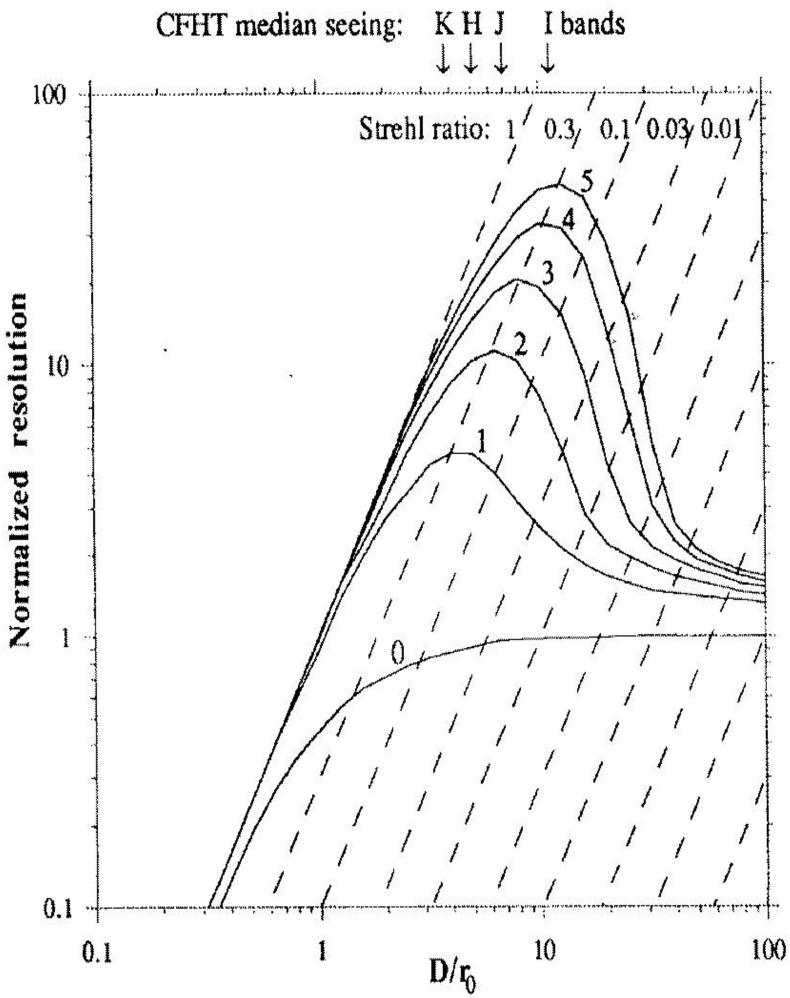
Total phase variance (without piston) $1.03(D/r_0)^{5/3}$.

How many modes to correct?

Residual rms phase error [rad^2] after correcting the first J Zernike modes (J>20):

$$\sigma^2 \sim 0.29\ (D/r_0)^{5/3}\ J^{-0.87}$$

Tip-tilt only: 0.134
Order 2 (J=6): 0.065
Max. gain for ~1rad^2 residual (Strehl ~0.3)



Measurements of turbulence

- Temperature: C_T^2 with micro-thermals or acoustic sounders (sonars)
- Image motion: affected by telescope shake/tracking
- Defocus (DIMM, any WFS)
- Scintillation (MASS)
- Optical profilers (SCIDAR, SODAR) need binary stars
- LGS profilers (on working multi-LGS AO systems)