Temporal response of RINGSS

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1 Introduction

This document considers temporal properties of scintillation signals in the ring-image turbulence sensor. This is needed to correct the signal variance for finite exposure time and to estimate the effective wind speed and the atmospheric time constant τ_0. This is an extension of such analysis done for the pupil-plane scintilation sensor. In both cases the signals a_m are the cosine and sine coefficients of angular frequency m, defined in the previous documents. Sum of their variances $S(m)$ is the angular power spectrum (APS) used to restore the turbulence profile.

The analysis is largely based on the work by Kornilov (2011) that introduces the quadratic approximation of APS dependence on the integration (exposure) time τ, valid when the wavefront shift during exposure, $V\tau$, is less than the scintillation scale. Here V is the wavefront speed which I call wind speed for clarity. The main expression from that work is

$$S_m(\tau) = S_m(0) - \frac{\pi\tau^2}{6} \int C_n^2(z) V^2(z) U_m(z) \mathrm{d}z \tag{1}$$

Here $C_n^2(z)$ is the turbulence strength (in $\mathrm{m}^{-2/3}$), $V(z)$ is the wind speed, and the special weighting functions $U_m(z)$ (hereafter U-functions) are analogous to the normal weighting functions (WFs) $W_m(z)$. The U-functions are computed similarly to WFs as integrals of the product of turbulence power spectrum and the square modulus of the spatial filters, but with additional multiplicative factor $\pi^2 f^2$. U-functions are measured in $\mathrm{m}^{-7/3}$.

Kornilov's theory operates with the variances of the signals and their covariances with a time lag of 1, assuming regular temporal sampling with cadence τ. Temporal covariances with larger lags are not needed. Let $C = \langle a_j a_{j+1} \rangle$ be the covariance with lag of one interval and $S = \langle a_j^2 \rangle$ the variance (assuming zero mean), then the correlation coefficient $\rho = C/S$ is the measure of the speed of signal variation. It is close to 1 for slow (e.g. well-sampled) signals, but can be small or negative when the signal varies faster than the sampling time. It follows that the signal variance for double exposure time $S(2\tau) = S(\tau)(1+\rho)/2$.

2 Software tools

The WF calculator `aweight2.pro` is modified to account for signal averaging by a linear blur of $V\tau$ meters and to return optionally the U-functions together with the normal WFs. A variant `rmassweight.pro` does a similar job for a quasi-Gaussian spectrum (in the monochromatic case both codes work similarly).

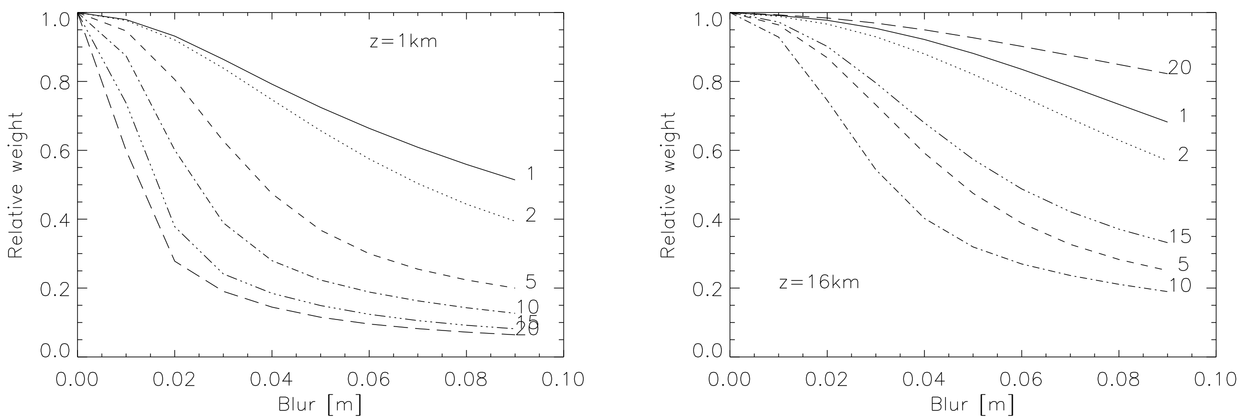

Figure 1: Reduction of the WFs caused by a spatial blur for turbulent layers at 1 km (left) and at 16 km (right). Numbers near he curves indicate the angular frequenchy m.

The code `rmasstemp.pro` computes the WFs without blur and with increasing amounts of spatial blur. Representative results are shown in Fig. 1 for one turbulent layer located either at 1 km or at 16 km distance. Calculation is done for spectral bandwidth $\Delta\lambda = 150$ nm using `rmassweight.pro`. For a low-altitude layer the decay is faster for larger m. For the high layer, this behavior is reversed at $m > 10$. This is likely a computational artifact caused by a spuriously large filter at low spatial frequencies. This behavior is not encountered in the monochromatic case, where the high-m WFS are not so strongly damped at large propagation distances. The effect of finite spectral bandwidth is important for the temporal analysis.

Analytic calculations of WFs help to define correction for the finite-exposure bias. The attenuation factor $\gamma_{12} = S(2\tau)/S(\tau) = (1 + \rho)/2$ can be computed from the measured signals, and the unknown factor γ_{01} (correction to zero exposure) can be estimated as $1-\gamma_{01} \approx 0.4(1-\gamma_{12})$ when the attenuation is not too strong: $\gamma_{12} > 0.6$ or $\rho > 0.2$. The approximation implies zero-exposure correction as

$$S(0) \approx S(\tau)/(0.8 + 0.2\rho). \tag{2}$$

This correction was established previously for the pupil-plane scintillation, so it is rather generic.

To test the analytical calculation of exposure-time effect, the simulator tool `ringsim.pro` now takes the wind speed as optional parameter and implements (also optionally) averaging of the simulated amplitude screens with a fixed amount of shifts to simulate blur. The signals a_m are related to the complex light amplitude in a non-linear way, so this implementation would work only in the weak-scintillation regime (as well as the weight calculation itself). The code `statmon.pro` is modified to return covariances together with variances. The code `testtemp.pro` implements the full cycle: simulation of atmospheric amplitude, creation of simulated data cube with `ringsim`, and its processing. The resulting variances and covariances are expressed in terms of WFs and compared to the analytic WFs. These simulations assume monochromatic light.

Figure 3 shows the results for a rather extreme wind speed of 10 and 20 m/s, i.e. a blur of 1-2 cm during 1-ms exposure. The simulated weights (squares) follow closely the analytical 1-ms weights

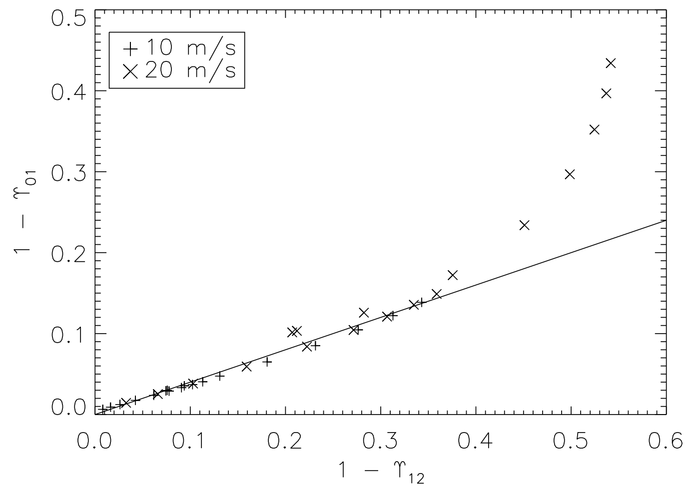

Figure 2: Signal atenuation ratio for double and single exposure γ_{12} is compared to the signal attenuation for single and zero exposure γ_{01} for spatial blurs of 1 cm (plus signs) and 2 cm (crosses). The line is $1 - \gamma_{01} = 0.4(1 - \gamma_{12})$.

(dashed lines). The correction to zero exposure (asterisks) works only when the correlation coefficient is positive, i.e. for $m < 14$ in the top panels and for $m < 6$ in the bottom panels. Therefore, experimentally measured correlation coefficients indicate the validity domain of the zero-exposure correction. This domain is wider than the validity range of the short-exposure approximation in Kornilov (2011), but still insufficient to use high-m signals for profile restoration under fast wind.

3 Estimation of the effective wind speed

The turbulence-weighted second moment of the wind speed V_2 is defined as

$$V_2^2 = J^{-1} \int C_n^2(z) V^2(z) \mathrm{d}z, \tag{3}$$

where $J = \int C_n^2(z)\mathrm{d}z$ is the turbulence integral. The parameter V_2 is uniquely related to the atmospheric AO time constant $\tau_0 = 0.31 r_0/V_2$, so its measurement is equivalent to measuring the time constant. However, the latter also depends on the turbulence integral J and wavelength, so here I use V_2 that has a clear physical sense of effective wind speed.

The difference between (3) and (1) is in the factors U under the integral. The idea of Kornilov is to combine several U-functions with coefficients C_k^U such that

$$\sum_k C_k^U U_k(z) \approx 1, \tag{4}$$

i.e. remove the dependence on the propagation distance z. Then eq. (1) can be transformed to get

$$V_2^2 \approx \sum_k C_k^U \Delta_k, \tag{5}$$

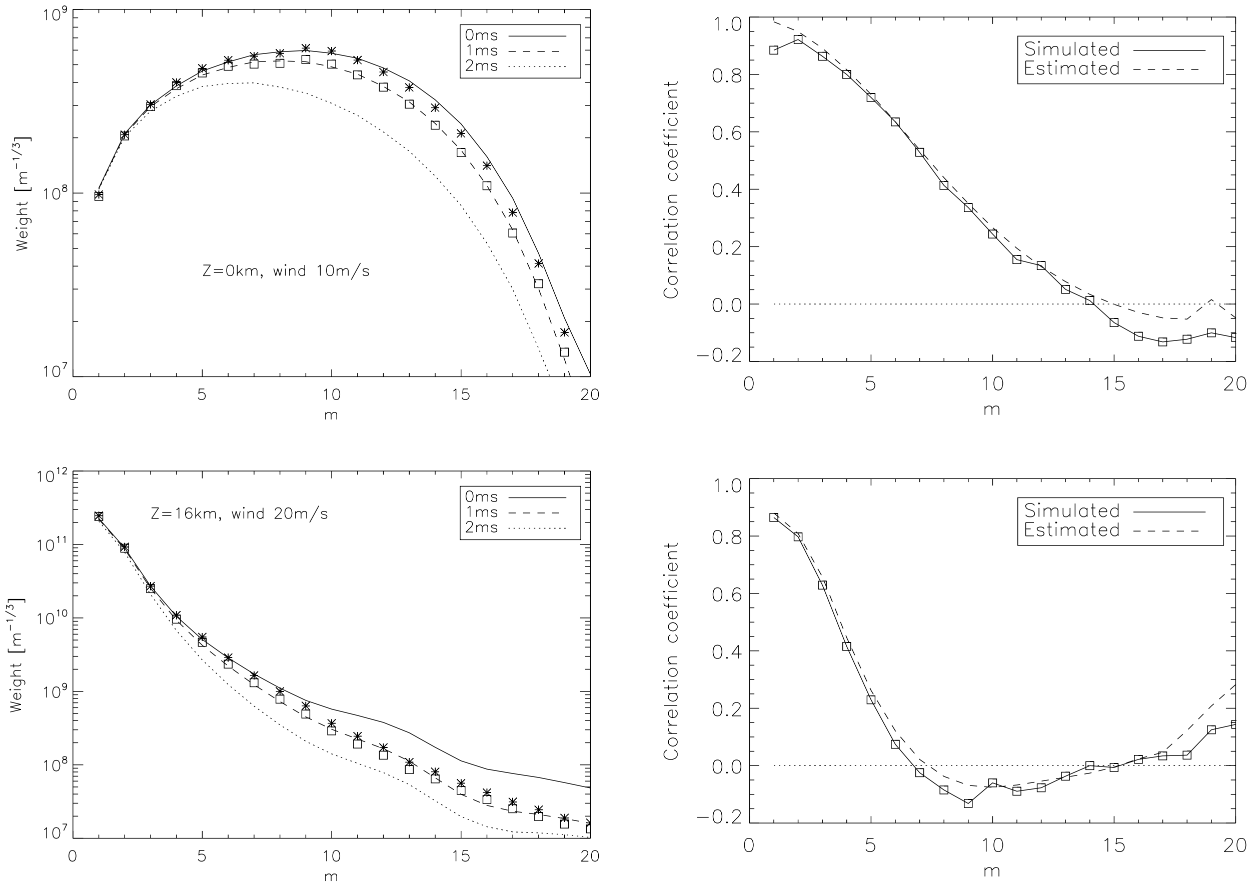

Figure 3: Results of `testtemp.pro` for a layer at the ground (top) and at 16 km (bottom). The left panels plot analytic WFs for exposure time of 0, 1, and 2 ms and the simulated WFs by squares (asterisks for corrected weights). The right-hand panels compare the simulated and analytic correlation coefficients.

where

$$\Delta_k = 6\frac{S_k(2\tau) - S_k(\tau)}{4\tau^2 - \tau^2} = S_k(\tau)(1 - \rho_k)/\tau^2 \tag{6}$$

The coefficients C^U can be found by solving eq. 4, following the Kornilov's recipe. The code `rmasstemp.pro` was used to find these coefficiens as the least-squares solution of eq. 4 on a logarithmically-spaced grid of z with weights proportional to z, using SVD to reject weak singular values in the matrix inversion. The first tests indicated that signals with m of 2 and 4 have very small coefficients C^U, while $m = 6, 7$ are the strongest contributors. Therefore, the set of U-functions used in the wind estimation can be restricted to $m = [1, 3, 6, 7, 8, 9]$. The corresponding coefficients C^U are $[9.8, 8.8, 20.2, 19.0, 17.5, 14.6] \times 10^{-15}$. These coefficients are benign in the sense of not increasing the measuremennt errors, so the resulting estimates of V_2 are robust. Figure 4 shows that the approximate response falls below one at $z < 1$ km (similarly to the MASS instrument), but not dramatically.

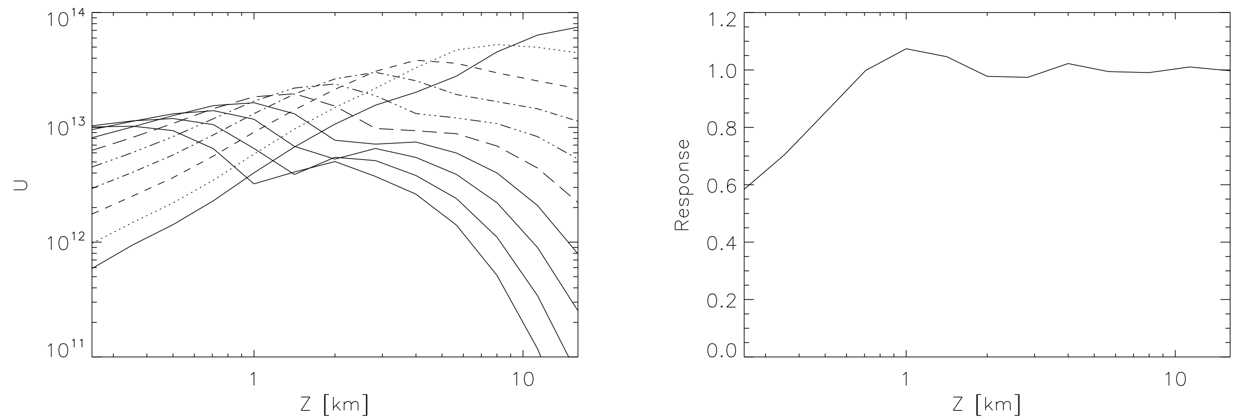

Figure 4: Left: set of U-functions for m from 1 to 10, right: approximation of z-independent response using this set. Results of `rmasstemp.pro`.

The code `profrest4.pro` takes variances and covariances as inputs to correct for the finite-exposure bias. The data used in the profile restoration now include the U-functions and the pre-computed coefficients C^U. Note that the WFs, U-functions, and coefficients depend on the instrument parameters and spectral response, they should be re-computed when these parameters change. Testing profile restoration on simulated data cubes that account for the finite exposure have demonstrated that the results are inaccurate when the exposure-time bias is too strong and the zero-exposure correction fails. To fix this problem, I restrict the maximum frequency used in the restoration to frequencies with non-negative correlation coefficients ρ_m. Referring to Fig. 3, this means $m < 14$ for the $z = 0$ case and $m < 7$ for the $z = 16$ km case. However, the minimum m is always set to 9 in order to get valid estimates of the effective wind speed.

The profile restoration and wind estimator were tested using again `testtemp.pro` that simulates atmospheric screens and data cubes for a single layer, single wind speed, and monochromatic light. The instrument parameters are defined in the `sim1.par` parameter file. The results show a very reliable estimate of the wind speed at $z > 1$ km; at smaller distances it is unter-estimated, as expected. The seeing measured from the simulated 1-second data cubes also matches the input seeing quite well, except the cases of strong scintillation, where the seeing estimator "over-shoots". The 16-km layer with 20 m/s wind gives the measured wind of 14.8 m/s while using signals with $m \leq 9$; the wind of 15 m/s is measured almost correctly at 12.3 m/s. The input seeing of $0.2''$ in both cases is faithfully recovered.

Figure 5 shows just one example of testing on simulated data. In this case, APS up to $m = 14$ is used for profile resoration, and the relative rms difference between data and model is 0.035. If the input seeing is increased to $2''$, the measured seeing becomes $2.35''$, i.e. an over-shoot caused by the departure from the small-signal regime.

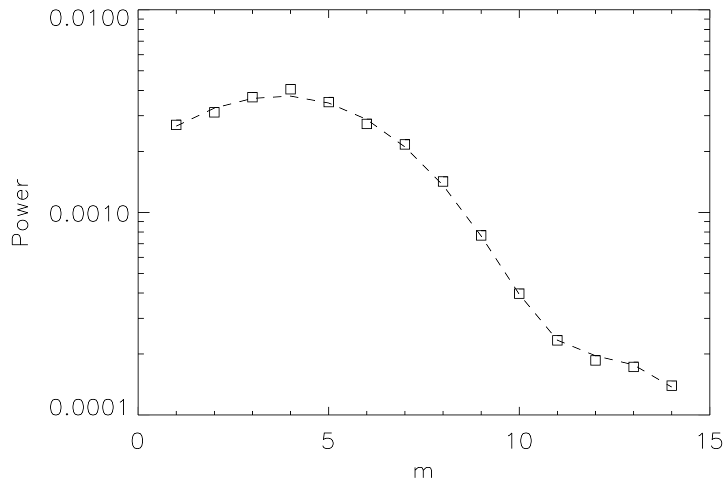

Figure 5: Measured APS (squares) and its fitted model (dash) for 1-km layer with 1″ seeing and 10 m/s wind. Measured seeing 1.06″, measured wind speed 9.1 m/s.

References

Kornilov, V. 2011, A&A, 530, 56