Investigation of turbulence profile restoration in RINGSS

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1 Introduction

The RINGSS turbulence profiler is based on the analysis of ring-like images of a single star to measure basic turbulence parameters, including a low-resolution profile (integrals J_j in 8 layers). Recently, RINGSS instruments have been compared to the regular MASS-DIMM monitor at Cerro Tololo and to the SCIDAR profiles at Paranal. Overall, a reasonable agreement has been found, with minor discrepancies. In this document, various aspects of the turbulence profile (TP) restoration in RINGSS are explored to find potential pitfalls and evaluate robustness. As typical to all single-star profilers, the dependence of the measured signals (statistical moments) on distance is smooth, so the TP restoration from the signals (an inverse problem) is intrinsically delicate. Minor details can cause appreciable differences in the results. The good news is that the integral characteristics (overall and FA seeing) are quite robust, and the discrepancies appear, mostly, in the distribution of turbulence between the layers.

2 Moving-layer simulation

Ability to discriminate between turbulent layers and to attribute them to correct altitudes has been tested with `testrestor2.pro`. It uses the parameter file of the real instrument. The angular power spectrum $S(m)$ is generated analytically using the weighting functions (WFs). A layer containing 70% of turbulent energy is placed at various heights, the remaining 30% are at 8 km. The APS is generated analytically (assuming zero wind) and the TP is restored. Figure 1 plots the ratio of the measured and input integrals (assuming $1''$ seeing). The measurement is always an under-estimate which increases with the height of the stronger moving layer. This is expected because the simulated APS does not account for the saturation, while saturation is corrected in the restoration.

The "waves" in the curve have a small amplitude of $\sim$10%. The minima correspond to situations where the moving layer is located between the fixed layers (e.g. at 0.35 km) and is distributed between them. When a $0.3''$ seeing is simulated, the curve does not drop at high altitude and stays very close to one, but the two local minima at 0.35 and 0.7 km still remain. The restored TPs evolve as expected, showing signal in only 2 or 3 layers and zero (or very little) elsewhere.

3 Variance of the APS

For a given data cube, the APS $S(m)$ is deduced from $N = 2000$ samples of the complex signal a_m (cosine and sine terms) at each frequency m. If the samples were statistically independent, the APS

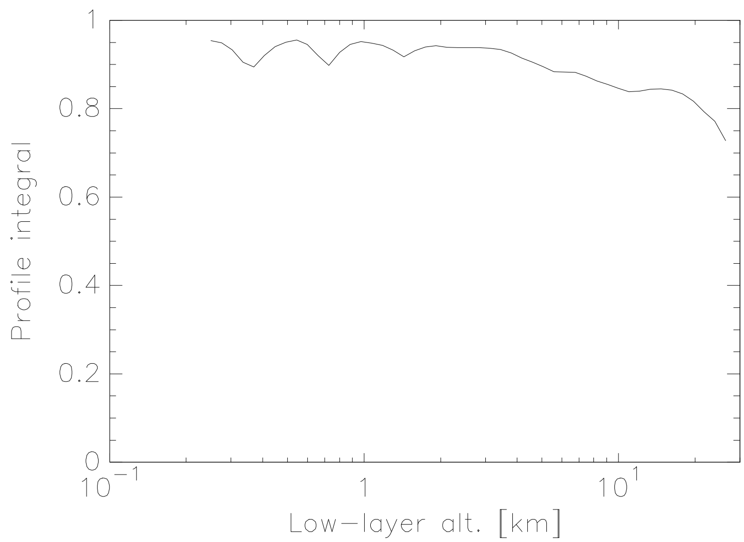

Figure 1: Ratio of measured and input total turbulence integrals in the moving-layer analytical simulation with a $1''$ total seeing and 30% in the fixed 8-km layer. The height of the strongest layer is variable.

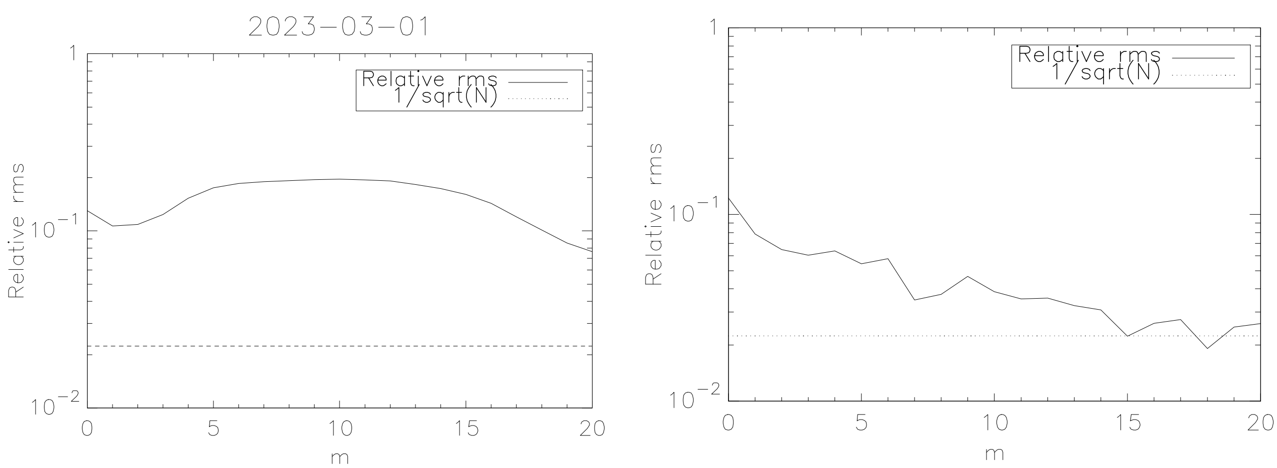

Figure 2: Relative rms fluctuations σ_s/S of the APS $S(m)$ vs. m. Left: determined from the Paranal data of 2023-03-01, right: simulated. In both plots, the theoretical floor $1/\sqrt{N} = 0.022$ is indicated by the dotted line.

would be determined with a relative error of $1/\sqrt{N} = 0.022$ (the distribution of $|a_m|^2$ is negative-exponential, square of its mean equals its variance). However, the signals a_m in consecutive frames of the cube are not mutually independent, particularly at low m, because the transit time of the scintillation pattern over the pupil is longer than the frame time. So, it is expected that the relative APS error at low m is larger than $1/\sqrt{N}$.

In each measurement cycle of RINGSS, 10 data cubes are acquired and processed. Variance

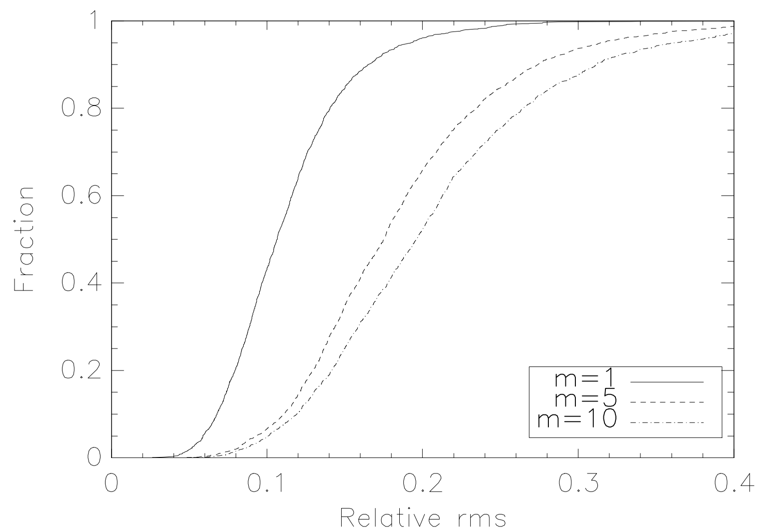

Figure 3: Cumulative distribution of the relative APS fluctuations at selected m for the night of 2023-03-01.

of the APS over each 10-cube sequence for a selected night (March 1, 2023) was computed using `momentvar.pro` by reading the `.stm` file. In Fig. 2, the median relative APS fluctuations for this night are plotted vs. m. Between $m = 5$ and $m = 15$, they are approximately constant at $\sim$0.2, an order of magnitude larger than the theoretical floor.

For comparison, I simulated 20 data cubes using `testvar.pro`. The simulations assume monochromatic light of 600 nm, $1''$ seeing distributed in the 0.7:0.3 proportion between 0.5 and 8 km layers, and a wind speed of 5 m/s. The photon and readout noise corresponding to a $V = 2$ mag star is simulated as well. The relative fluctuations of the simulated APS are plotted in the right panel of Fig. 2. They show the expected behavior, decreasing with m and approaching the theoretical floor. The relative rms in the simulation match the real data only at $m = 0$ and $m = 1$.

Returning to the real data, the cumulative histogram of the relative APS fluctuations (for 3 selected m) is plotted in Fig. 3. The distributions for $m = 5$ and $m = 10$ are similar. Large difference between the measured and simulated APS fluctuations can have only one explanation — a real change of the turbulence properties between the 2-s data records. So, the classical representation of turbulence by a random stationary process is at odds with reality, the actual turbulence has strong intermittency at small spatial scales (large m). The intermittency is strongest in the lowest layers. The high layers that contribute mostly to the small-m signals behave better. However, these signals come from a larger air volume because of averaging along the line of sight and in the transverse direction (faster wind at high altitudes).

This phenomenon has been noted before by V. Kornilov who explored variance of statistical moments in MASS computed from the 1-s data (1000 samples per second). He argued that for a better modeling of the signal, the TPs should be restored from the individual 1-s moments and then averaged over 60 s. This approach has been implemented in the latest version of the MASS software. However, TP restoration from the moments is a non-linear operation, so the average TP restored from 1-s mo-

Table 1: Ratio of mean turbulence integrals python/IDL

$m_{\max}$	0	1	2	3	4	5	6	7
$m_{\max} = 15$	0.994	1.022	1.008	0.996	0.670	1.036	1.055	0.888
$m_{\max} = 17$	1.122	0.217	1.176	1.037	0.634	1.469	1.025	0.873

ments differs systematically from the TP restored from the 60-s average moments. The difference is manifested, mostly, by the non-zero turbulence values in all layers, generated by the TP restoration algorithm from the noise (it acts like a "rectifier"). In contrast, the standard TP restoration based on the 60-s average moments distributes the turbulence in fewer layers, with zeroes in the remaining layers.

The relation between the statistical moments $S(m)$ and the TP J_j is linear. So, the TP restored from the *average* $S(m)$ corresponds to the average conditions during each measurement period and avoids the noise-rectification bias. This classical approach is adopted in RINGSS. In consequence, zero values of J_j (i.e. layers without turbulence) are frequent in the RINGSS data.

4 Under-estimation of the 0.25-km layer?

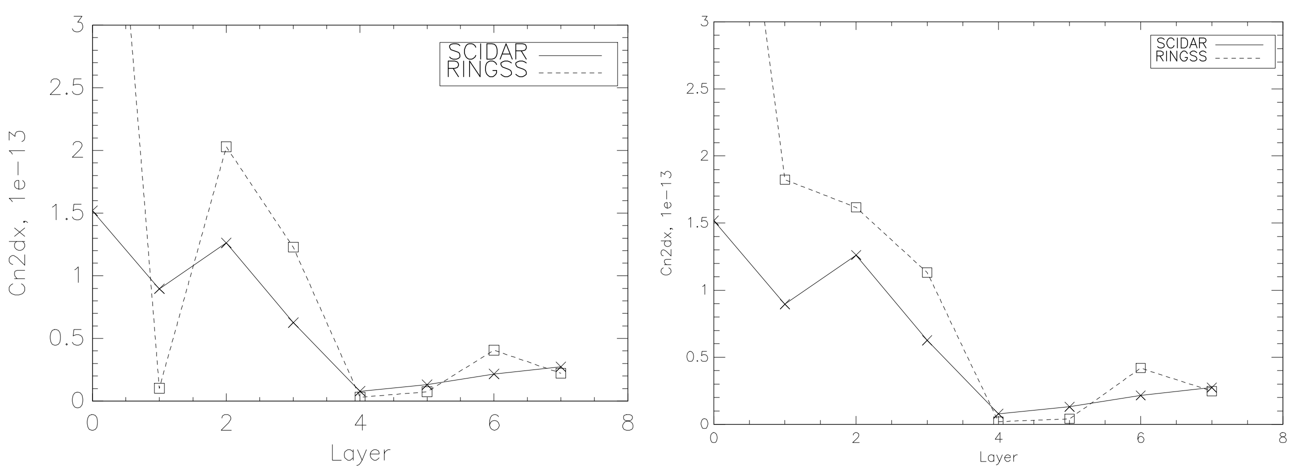

Figure 4: Mean turbulence profiles measured by RINGSS and SCIDAR at Paranal (the latter is matched to the RINGSS layers). Left: profiles restored with the python code, $m_{\max} = 15$; right: IDL profles with $m_{\max} = 17$.

Turbulence profiles measured by RINGSS at Paranal were matched in time and vertical resolution to the SCIDAR profiles and compared. There is an excellent qualitative agreement, showing turbulence in the same layers at the same time. However, comparison of the average matched integrals in the layers (Fig. 4) reveals that RINGSS under-estimates the 0.25-km layer 1 and re-distributes its turbulence between adjacent layers 0 and 2. When the TP is restored with IDL using two additional frequencies, up to $m_{\max} = 17$, this effect disappears, while the layers 3 and higher are not affected. With the default $m_{\max} = 15$, the python and IDL codes produce very similar profiles. This illustrates the delicate nature of the TP restoration, investigated further below. Table 1 shows the ratios of mean integrals J_1 in the python and IDL versions of the TP restoration for two $m_{\max}$. The good agreement for $m_{\max} = 15$ is documented, while for $m_{\max} = 17$ the 0.25-km layer becomes, on average, almost 5 times stronger. The non-unit ratio in layer 4 is not a concern because turbulence in this layer was very small.

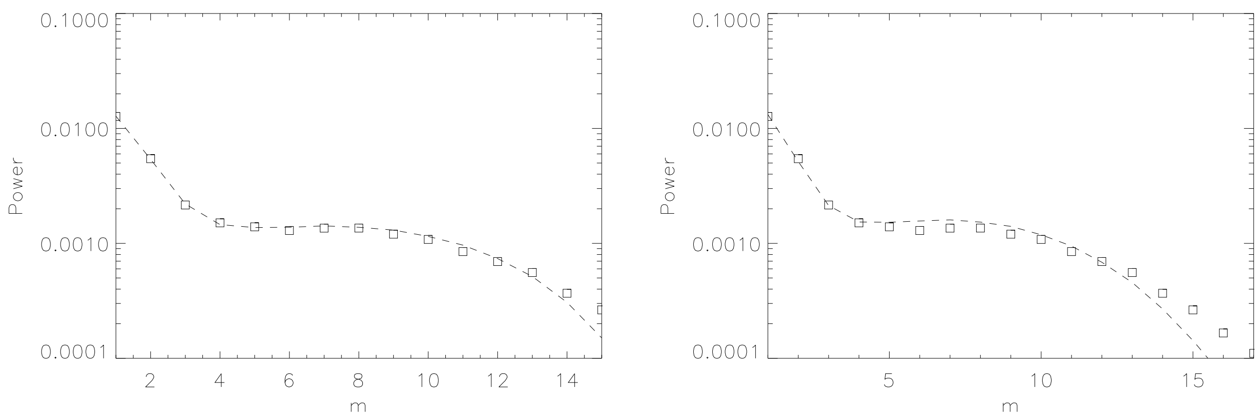

Figure 5: Turbulence profile restoration for the sample #950 (2023-03001T05:04:36) with $m_{\max} = 15$ (left) and $m_{\max} = 17$ (right). Squares are the APS, dashed lines are their fitted models.

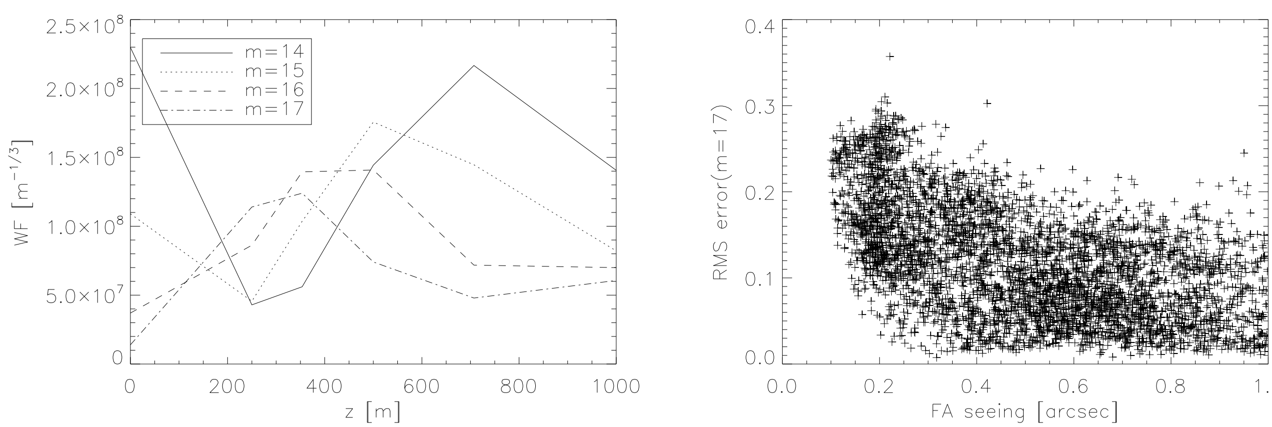

Figure 6: Left: WFs at low z. Right: rms fitting error vs. FA seeing for Paranal data with $m_{\max} = 17$.

Increase of $m_{\max}$ is associated with a worse fits of the APS models. I selected for the detailed study one record #950 from the Paranal data with a large fitting error (total seeing 1.27″, FA seeing 0.18″). Figure 5 shows the APS models (dashed lines) and the data (squares, noise-subtracted and corrected for biases). The model fails to match the APS at large m and calls for lower values, which is the cause of the increased fitting error. Table 2 lists the rms fitting errors and the TPs J_j for this

Table 2: TPs for the sample #950 (J_j in 10^{-13} $\mathrm{m}^{1/3}$)

$m_{\max}$	rms	0	1	2	3	4	5	6	7
		0	0.25	0.5	1	2	4	8	16
15	0.13	9.57	0	0.23	0	0.02	0	0.19	0.24
16	0.21	9.63	0	0.35	0	0	0	0.18	0.25
17	0.27	8.21	1.60	0	0	0	0	0.10	0.29

particular record for three values of $m_{\max}$. The total and FA seeing remain practically unaffected, but with $m_{\max} = 17$ turbulence appears in the 0.25-km layer 1 at the expense of the adjacent layers (without change of the total integral).

Figure 6, left, plots the WFs for relevant m at low altitudes to better understand how they discriminate between the layers. With m increasing from 14 to 17, their local maximums shift to smaller z. The right panel shows an anti-correlation of the fitting errors for $m_{\max} = 17$ with the FA seeing. Large fitting errors are associated with poor modeling of the low layers, when they dominate the overall seeing and the FA seeing is small. Conversely, when the higher layers are strong, the fitting becomes better, independently of $m_{\max}$.

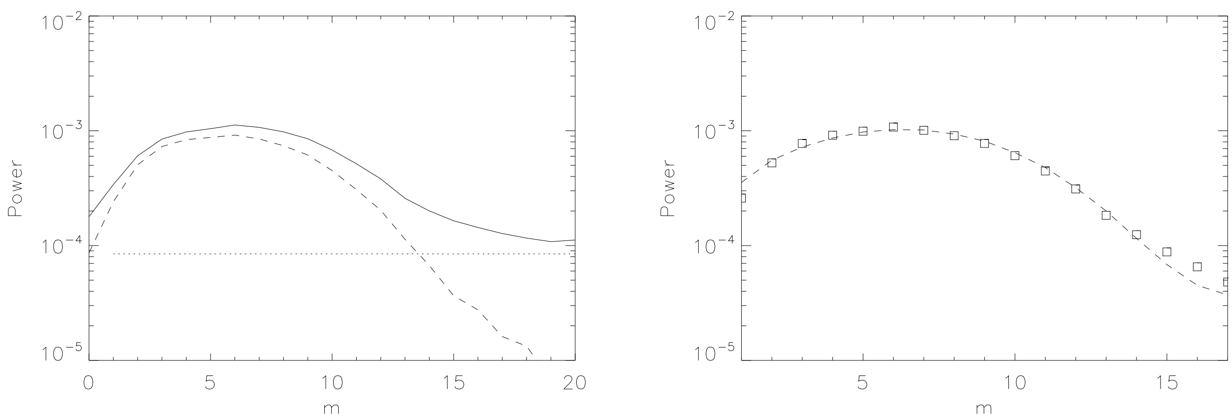

Figure 7: Simulations: layers at 0 and 0.35 km in a 0.6:0.4 proportion with 1″ total seeing and 5 m/s wind. Left: power, covariance, and noise level vs. m. Right: data (squares) and fitted model (dash) for $m_{\max} = 17$.

The issue was tested by simulation with `testsimul-low.pro`, a version of the general end-to-end simulation package. Two turbulent layers at the ground and at 0.25, 0.35, or 0.5 km were simulated with a 0.6:0.4 proportion and a 1″ total seeing, a star of $V = 2$ mag, a wind of 5 m/s, and monochromatic light (for low layers, spectral bandwidth is not important). The simulated cube was processed in the standard way and the TP was restored with both $m_{\max} = 15$ and $m_{\max} = 17$. As in the real data, $m_{\max} = 17$ increases the fitting error (only slightly) and the 0.25-km layer intensity (even when the turbulence was actually at 0.5 km). Otherwise, the restored TPs are a reasonable match to the input TPs, and no under-estimation of the 0.25-km layer is observed. So, we cannot say that the choice of $m_{\max}$ is critical for correct measurements of low layers. The "tails" of the APS at large m, as seen in Fig. 5, are not reproduced by the simulations.

The APS tails at large m could be caused by an under-estimated noise. Indeed, by artificially increasing the noise level by a factor of 1.05 to 1.1, we can reduce the fitting error. However, such an increase is not supported by the full data: the minimum ratio of APS at $m = 20$ (almost entirely noise) and the estimated noise is 1.00, the mean ratio under good seeing ($< 0.6''$) is 1.05. Furthermore, in most cases (including the sample #950) signal at $m = 17$ substantially exceeds the noise. A hypothesis that the increased noise could be caused by the spatially non-uniform detector sensitivity (we do not correct for flat field) is refuted because even under very good seeing the estimated noise matches well the APS level at $m = 20$.

The high-frequency tail of the APS is likely produced by some un-modeled effect. For example, localized small-scale optical defects in the telescope would produce small details in the ring image which will be modulated by the scintillation and will contribute to the signal at large m (a kind of cross-talk between low and large m). Such small-scale features are indeed seen in the average ring images.

It must be noted that the python and IDL codes differ in the estimated wind speed systematically: the mean ratio of wind speeds python/IDL is 0.75, most individual values are between 0.5 and 0.9. This is related to the difference in the U-functions and their linear combinations between both codes — another delicate aspect of all single-star profilers.

5 Summary

This study highlights limits of the TP restoration from scintillation of single stars in the RINGSS instrument.

1. The real data show that turbulence is highly intermittent because differences of the APS values derived from successive 2-s image cubes substantially exceed expectations from the standard random stationary turbulence model.

2. In idealized simulations, the restoration algorithm correctly interprets signals from two layers, like in MASS. This is also true for more realistic end-to-end simulations including noise, wind, finite pixels, etc.

3. Comparison between RINGSS and SCIDAR indicates that the 0.25-km layer can be systematically under-estimated by RINGSS, distributing its signal among adjacent layers. This can be "fixed" by using higher angular frequencies (up to $m = 17$), but the APS shape is still not well-modeled. The "tails" of the APS cannot be reproduced by simulation. They could be caused by some unaccounted for effect, such as small optical defects of the telescope. The $m = 17$ frequency corresponds to the angular period of 12.17° and the linear period of 1.8 cm on the pupil (at 5 cm radius).

The number and heights of the layers can be flexibly assigned in the TP restoration software. The questionable 0.25-km layer can be eliminated, thus "solving" (in fact, hiding) the problem. The TP values at 0.25 km delivered by RINGSS should be considered with some reservation. The integral parameters (total and FA seeing) are robust.