Calibration of the MASS time constant by simulation

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1 Introduction

The adaptive optics atmospheric time constant τ_0 is defined as

$$\tau_0 = 0.314 r_0/\overline{V} = 0.057\ \lambda_0^{6/5} \left[\int_o^\infty C_n^2(h)\ V^{5/3}(h)\ \mathrm{d}h \right]^{-3/5}, \tag{1}$$

where $C_n^2(h)$ is the vertical profile of the refractive index structure constant, $V(h)$ is the vertical profile of the modulus of the wind speed. In the following we always assume that τ_0 refers to the wavelength $\lambda_0 = 0.5\ \mu$m.

The MASS instrument implements an approximate method of estimating τ_0 from the *differential-exposure scintillation index*, DESI. DESI is computed for the smallest 2-cm MASS aperture as a differential index between 1 ms and 3 ms exposures. It has been shown in [1] that DESI is useful for estimating the atmospheric time constant. A formula

$$\tau_{MASS} = 0.175\,\mathrm{ms}\ (\sigma^2_{DESI})^{-0.6} \tag{2}$$

has been suggested on the basis of limited data on real turbulence profiles. This formula is implemented in the standard MASS data processing

It has been found by means of simulations that τ_{MASS} calculated from (2) needs a corrective coefficient around $C = 1.27$. It was suggested that the true time constant can be derived from the MASS data by applying this corrective factor and adding the contribution of the ground layer which is not sensed by MASS, but can be measured with MASS-DIMM. Therefore, the AO time constant is estimated as

$$\tau_0^{-5/3} = (1.27\,\tau_{MASS})^{-5/3} + (0.057)^{-5/3} \lambda_0^{-2}\ V_{GL}^{5/3}\ (C_n^2 \mathrm{d}h)_{GL}. \tag{3}$$

The intrinsic accuracy of such estimate was evaluated to be ±20% or better.

Travouillon et al. [3] have derived a somewhat different correction factor $C = 1.73$ by calculating τ_0 from the turbulence profile measured by MASS and using NCEP wind velocities. Even larger factors of 2.45 and 2.11 were determined in [2] by the same method. This prompted a re-investigation of this matter by doing new simulations.

2 Simulation method

We simulate one phase screen at a given distance z from the instrument and calculate the intensity at the ground by means of the program `simatmpoly.pro`. A set of wavelengths is used, with the resulting intensity being a weighted sum for all wavelengths. Here we approximate the spectral response of MASS by 4 wavelengths of [400,450,500,550] nm with weights [0.31, 0.885, 0.60, 0.27]. This should mimic the response of TMT MASS-DIMMs, as studied by Kornilov [4]. He found for these instruments effective wavelength 474 nm and the bandwidth 99 nm (response curve `without.crv`). For our 4-wavelength approximation, the effective wavelength and bandwidth are 470 nm and 116 nm, respectively.

The simulated intensity distributions are saved in a binary file. In the previous simulator `simmass3.pro` they were used in a Monte-Carlo approach where the 2-cm MASS aperture was "dragged" through the intensity screen, while 1-ms and 3-ms exposure time was emulated by suitable blurring of the aperture in one direction and by 3x binning. Here we take a more direct approach and calculate DESI with the spatial filter

$$P(f_x) = \mathrm{sinc}(f_x b) - \mathrm{sinc}(3 f_x b), \tag{4}$$

where f_x is the component of spatial frequency, $b = V t_{exp}$ is the blur in the x-direction caused by the wind speed V during exposure time $t_{exp} = 1$ ms and $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. The circular aperture of diameter d implies filter

$$A(f) = \frac{2 J_1(\pi f d)}{\pi f d}. \tag{5}$$

The energy spectrum of the intensity is multiplied by the combined filter $(AP)^2$ and summed over all frequencies to get the DESI index. By omitting all filters, we obtain the raw scintillation index, by using only filter A – the scintillation index in the 2-cm aperture. The program is `simmass4.pro`. Results for a test case were compared with previous simulations and found to be in agreement. This validates the code.

Considering that the calculation of intensity distribution is the most time-consuming task, we simulate the intensity screen for a given seeing and propagation distance and then calculate DESI and τ_{MASS} for a set of 12 wind speeds, from 10 m/s to 65 m/s, by changing only the filter P. The calculation is repeated for 3 distances to the layer, 5, 10, and 15 km, and for 5 values of seeing ε, from $0.3''$ to $1.5''$. Therefore we cover a wide range of conditions, with a varying degree of saturated scintillation. The largest scintillation index is 0.94 (layer at 15 km, seeing $1.5''$). In each case, the true time constant (at 500 nm) is known, $\tau_0 = 0.31 r_0/V = 0.31(0.101/\varepsilon)/V$. We determine the corrective coefficient $C = \tau_0/\tau_{MASS}$ for each pair of time-constant values.

3 Results

Figures 1 to 3 show the correction factor C versus wind speed. For both low and high wind speed, MASS over-estimates the time constant ($C < 1$), while for mid-range wind speeds, more typical for the high atmosphere, it under-estimates it. The correction factor depends on the layer distance and, to a smaller extent, on the turbulence intensity (seeing). This last dependence is caused by saturation of scintillation. The saturation increases C for slow wind and decreases it for fast wind. Compare this to Fig. 1 of [2], where a large dependence on the wind speed and altitude is illustrated by calculating DESI in the linear weak-turbulence approximation (no saturation).

Figure 4 shows the comparison between τ_0 and τ_{MASS} concentrating on the important region $\tau_0 <$ 5 ms (as we saw, MASS strongly over-estimates time constant when it is large). A line $\tau_{MASS} = \tau_0/C$ with $C = 1.27$ gives a general sense of the relation, but no exact value of the "correction factor" can be determined. Sequences of points located above this line in "arcs" correspond to strong turbulence and fast wind, when MASS again over-estimates the time constant.

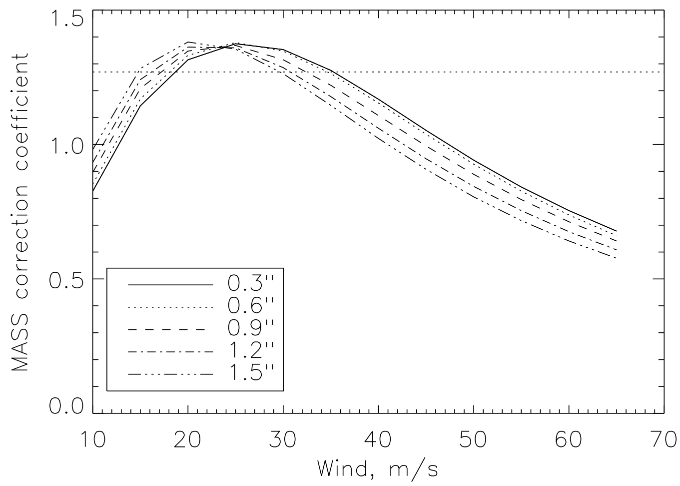

Figure 1: Correction factor C as a function of the wind speed for a single turbulent layer at distance 5 km from MASS. The turbulence strength of the layer (seeing) is from $0.3''$ to $1.5''$. The dotted horizontal line shows $C = 1.27$.

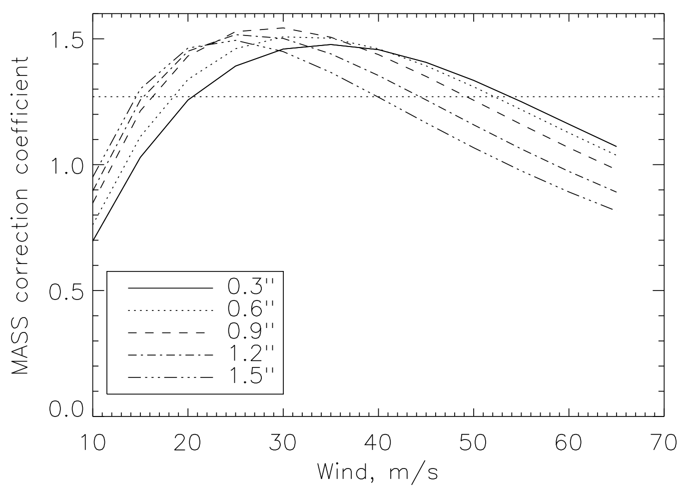

Figure 2: Correction factor C as a function of the wind speed for a single turbulent layer at distance 10 km from MASS. The turbulence strength of the layer (seeing) is from $0.3''$ to $1.5''$.

4 Discussion

Typical atmospheric time constant $\tau_0 = 2$ ms corresponds to the wind speed of 15.5 m/s under $1''$ seeing or to 31 m/s under $0.5''$ seeing, more typical for the high atmosphere. Strong turbulence is usually associated with jet stream at altitude around 12 km above sea level or at ~ 10 km above observatory. Observations at 45° above horizon place such turbulence at 14 km from the instrument. Therefore, typical conditions correspond to $V \sim 30$ m.s and propagation distances from 10 km to 15 km. As can

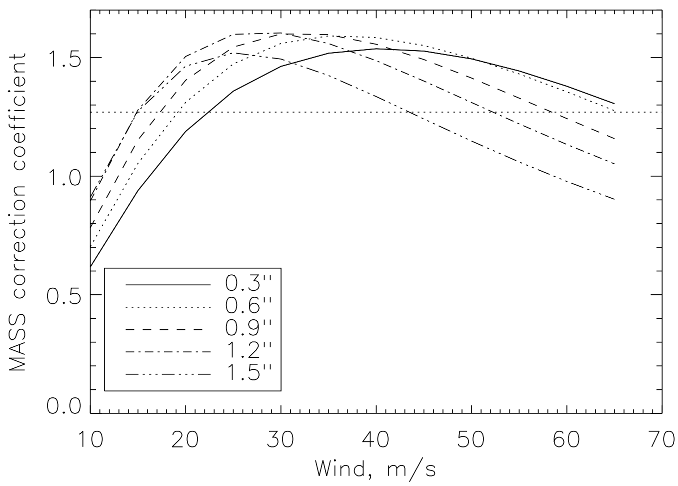

Figure 3: Correction factor C as a function of the wind speed for a single turbulent layer at distance 15 km from MASS. The turbulence strength of the layer (seeing) is from $0.3''$ to $1.5''$.

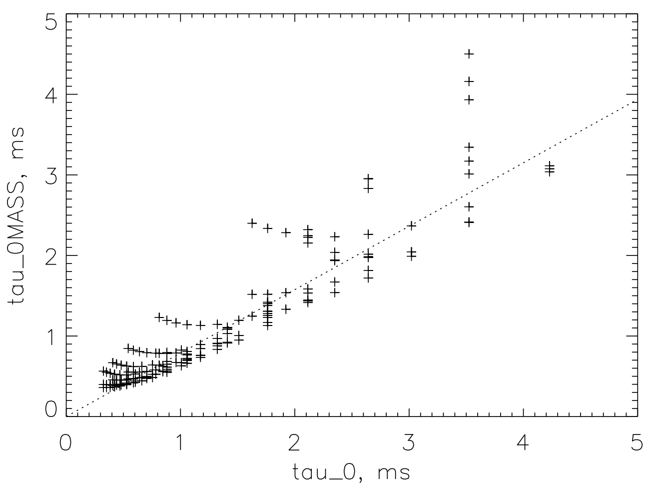

Figure 4: All values of τ_{MASS} plotted against τ_0. The dotted line shows previous correction factor $C = 1.27$.

be seen in Figs. 2 and 3, a correction factor $C \sim 1.45$ is more suitable in this regime than $C = 1.27$.

No single value of C can be proposed because the ratio τ_0/τ_{MASS} depends on the profile of turbulence and wind. In the extreme cases, this ratio can be as low as 0.6 or as high as 1.6. This range shows the approximate nature of the MASS method to estimate τ_0. It has been estimated in [2] that for a set of realistic profiles the rms scatter of the ratio τ_0/τ_{MASS} is about 20% around its mean value of 1.27 or so. The average correction factor is sensitive to the spectral response of MASS, so different values should be used depending on the MASS instrument and its feeding telescope.

The correction factor $C = 1.73$ proposed by Travouillon et al. [3] is clearly too large, as in all studied cases we find $C < 1.6$. The reason for this discrepancy is, apparently, in the use of NCEP wind velocities.

It was clear right from the start that the method of estimating τ_0 from DESI could be improved. It was suggested in [1] to increase the sampling time in the case of slow turbulence in order to remove the bias. Another option would be to involve temporal covariances of signals in multiple apertures. Unfortunately, DESI with indices binned data or temporal cross-covariances are not calculated by the MASS software `Turbina`, making it impossible to apply these future techniques to the existing data. However, considering that the height and intensity of the dominant turbulent layer is known from the MASS turbulence profile and that the behaviour of the MASS bias is understood, an *a posteriori* correction can be developed to reduce the scatter of C.

References

[1] Tokovinin A., 2002, "Measurement of seeing and atmospheric time constant by differential scintillations," Appl. Optics, 41, 957

[2] Tokovinin A., 2006, "Calibration of the MASS time constant measurements". Internal report, June 22, 2006. `http://www.ctio.noao.edu/ atokovin/profiler/timeconst.pdf`

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[4] Kornilov V. The verification of the MASS spectral response. September 14, 2006 `http://www.ctio.noao.edu/ atokovin/profiler/mass_spectral_band_eng.pdf`