Calibration of the MASS time constant measurements

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1 Introduction: MASS time constants

The atmospheric time constant τ_0 is an important parameter for all high-resolution techniques, especially adaptive optics. The definition of the AO time constant is

$$\tau_0 = 0.314 r_0/\overline{V} = 0.057\ \lambda_0^{6/5} \left[\int_o^\infty C_n^2(h)\ V^{5/3}(h)\ \mathrm{d}h \right]^{-3/5}, \tag{1}$$

where $C_n^2(h)$ is the vertical profile of the refractive index structure constant, $V(h)$ is the vertical profile of the modulus of the wind speed. In the following we always assume that τ_0 refers to the wavelength $\lambda_0 = 0.5\ \mu$m.

The differential-exposure scintillation index (DESI) is computed for the smallest 2-cm MASS aperture as a differential index between 1 ms and 3 ms exposures. Instead of binning the signal in 3 ms, the DESI index σ^2_{DESI} is actually computed as

$$\sigma^2_{DESI} = 2/9\ (3\sigma_0^2 + \sigma_2^2 - 4\sigma_1^2), \tag{2}$$

where σ_i^2 is the covariance of the normalized light intensity with a time lag of i sampling periods. The first term σ_0^2 is a normal scintillation index with the photon-noise bias subtracted.

It has been shown in [4] that DESI is useful for estimating the atmospheric time constant. A formula

$$\tau_{MASS} = 0.175\,\mathrm{ms}\ (\sigma^2_{DESI})^{-0.6} \tag{3}$$

has been suggested on the basis of limited data on real turbulence profiles.

The empirical calibration coefficient $K = 0.175$ ms is used in the actual calculation of the MASS time constants τ_{MASS}. However, the real instrument is different from the conditions used to derive K: the wavelength response of MASS is broad-band centered at $\lambda \approx 0.45\ \mu$m, the aperture diameter is about 2 cm and the aperture is conjugated to the ground, not to -1 km.

In this Report, the calibration of the MASS time constant is re-considered. First, we use the same method as in [4] but with the real instrument parameters and a larger test set of profiles. Second, we perform direct numerical simulations of MASS with a single phase screen, taking into account the effects of saturated scintillation. Finally, some other data are involved to check the MASS time constants.

2 Calibration with real turbulence profiles

The response of the MASS DESI to a single turbulent layer at a distance (range) h moving with the speed V is described by the corresponding weighting function $W_{DESI}(h,V)$, so that for j layers

$$\sigma^2_{DESI} = \sum_j W_{DESI}(h_j, V_j)\ (C_n^2 \mathrm{d}h)_j. \quad (4)$$

Comparing this with Eq. 1, we see that MASS would give an exact measure of τ_0 if a condition $W_{DESI}(h,V) \propto V^{5/3}$ holds. For the real MASS DESI signal, this condition can be only approximate. How good is it?

We computed the DESI weight using the recipe of [4] modified for a poly-chromatic spectral response according to [5]. A central wavelength 450 nm and FWHM bandwidth 100 nm were assumed, mimicking the actual instrument characteristics. The exposure time is 1 ms and aperture diameter d = 2 cm. The ratio $W^{0.6}_{DESI}(h,V)/V$ is plotted in Fig. 1. Its deviation from a constant can be directly interpreted as a bias in τ_{MASS} for a single turbulent layer. The plots show that MASS

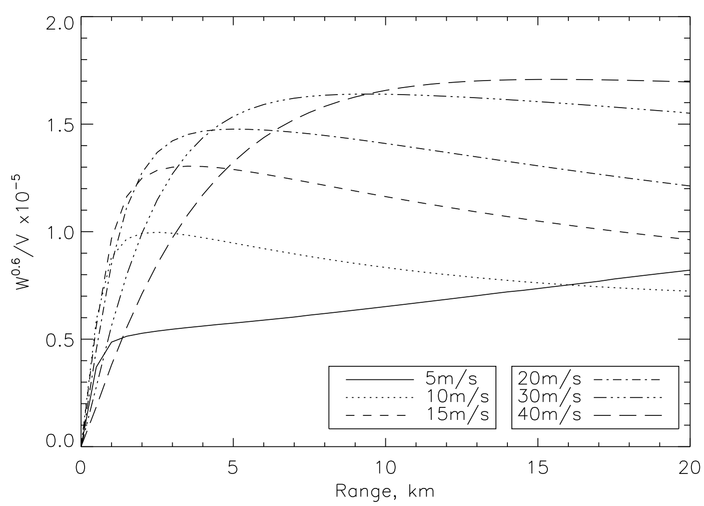


Figure 1: Plots of $W^{0.6}_{DESI}/V$ vs. range for different wind speeds.

under-estimates the contribution of turbulence below 1-2 km to the integral (hence over-estimates τ_0). Low wind speeds also lead to a strong bias, but for typical wind speeds in the range 15–50 m/s the condition $W^{0.6}_{DESI}(h,V)/V = const.$ roughly holds for the high atmosphere, to within ±20%. The intrinsic accuracy of τ_{MASS} should be of the same order.

The profiles of $C_n^2(h)$ and $V(h)$ at Cerro Pachón have been measured with balloons in 1998 during the Gemini site-testing campaign [1]. These data were retrieved for the study of the Gemini GLAO system. Of the 44 profiles, only 26 contain the wind speed data and are useful for the present study.

For each profile, the time constant τ_0 was calculated. Also, the constant $\tau_{0,h}$ ignoring the contribution of layers below 1 km was calculated, in hope to be more directly comparable to the τ_{MASS}. The bias is evaluated in the logarithmic sense by computing the average of $r = \log_{10}(\tau_{MASS}/\tau_0)$ and the rms scatter σ_r.

The comparison of τ_{MASS} with τ_0 and $\tau_{0,h}$ is shown in Fig. 2. Most of the time, MASS slightly under-estimates the true time constant. However, for low wind speeds or strong ground-layer turbu-

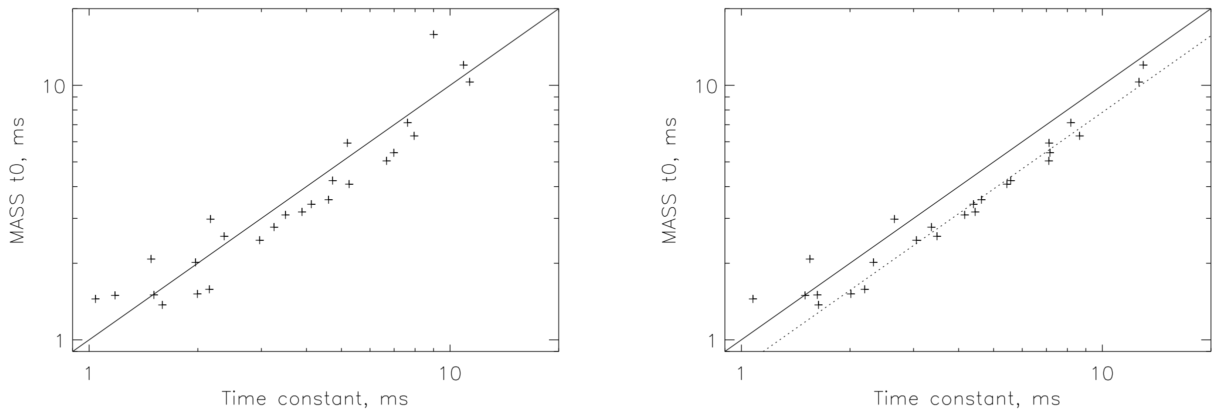


Figure 2: Test of the MASS τ_0 measurements against 26 real turbulence profiles for Cerro Pachón. The horizontal axis shows true time constants computed for the whole atmosphere (left) and above 1 km (right). The full line corresponds to equality, the dotted line is the estimated MASS bias of 0.78.

lence the MASS bias is positive, and on the average the ratio τ_{MASS}/τ_0 is close to one. By restricting the comparison only to high layers and high wind speeds (23 profiles out of 26), a more sound estimate of the MASS bias is obtained, $\tau_{MASS}/\tau_0 = 0.78$.

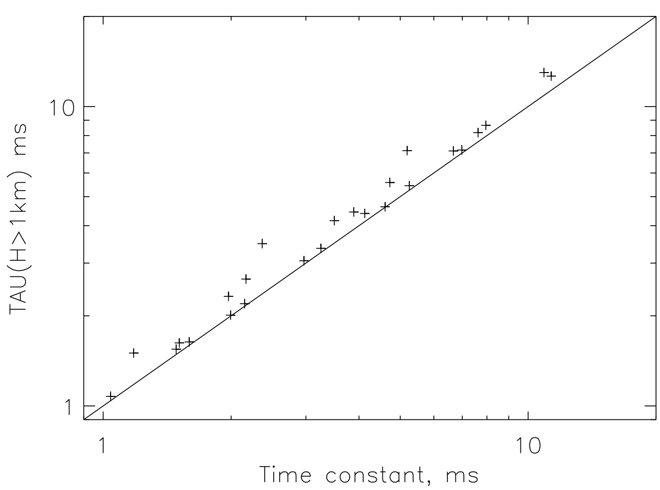
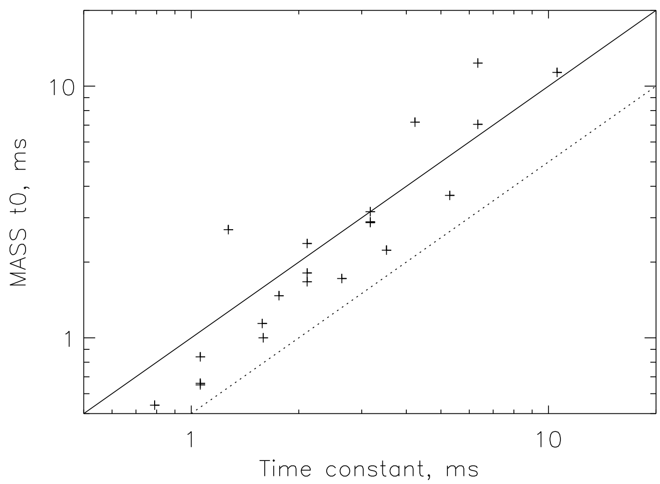


Figure 3: Comparison of the truncated estimate $\tau_{0,h}$ (vertical axis) with the true time constant τ_0 (horizontal axis). The full line corresponds to equality.

Figure 3 compares the true and truncated time constants. It shows that the time constant is mostly determined by the high atmospheric layers and the bias caused by truncation is small, about 1.16 on the average.



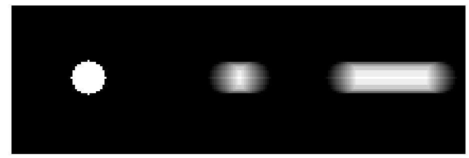


Figure 4: Comparison of the τ_0 for a single-layer turbulence and its simulated measurements by MASS. The full line corresponds to equality, the dotted line is 2 times lower. The images of the 2-cm aperture blurred by the 20 m/s wind in 1 ms and 3 ms exposures are shown on the right.

3 Numerical simulation

A scintillation signal produced by a single turbulent layer has been simulated numerically. The method of the simulation is described in [6]. The pixel size of the simulated phase screens is 5 mm, with 1024^2 pixels (5.12 m). The light was monochromatic, $\lambda = 450$ nm. To simulate the effect of the finite exposure time of $t = 1$ ms, the MASS aperture with $d = 2$ cm was blurred by a liner shift Vt. A blur of $3Vt$ simulates the signal binned in 3 ms (Fig. 4), so a DESI index can be computed in the same way as the differential index between two blurred apertures. Alternatively, the DESI was computed directly from the 1-ms samples using Eq. 2. These two alternative estimates of DESI are in a very good agreement.

This simulation takes into account the effects of strong (semi-saturated) scintillation. A single layer with seeing of 0.3″, 1.0″ and 1.5″ was placed at $h = 10$ km. Also layers with (0.5″, 2 km) and (1.0″, 4 km) were simulated. The scintillation index ranged from 0.05 to 0.86. Each layer was given a range of wind speeds from 5 to 50 m/s.

The results of all simulations combined are plotted in Fig. 4. Of the three cases of high positive bias of τ_{MASS}, two correspond to the slow wind speed 5 m/s and one to $V = 50$ m/s and $h = 2$ km. Ignoring these cases, the remaining 19 points lead to the average MASS bias of $\tau_{MASS}/\tau_0 = 0.79$. This is very close to the result of the previous Section. To remove the bias, the calibration coefficient in (3) should be set to $K = 0.22$ ms.

4 Additional data

Suzanne Kenyon extracted the MASS data for Cerro Tololo for the period April 2003 to April 2006 (64504 points) and matched them to the data on wind speed extracted from the NCEP/NCAR database. These meteorological data are available on a coarse vertical grid with a time sampling of 6 h. The matching in time consists in selecting the V data closest in time to the MASS data. The matching in altitude was done in two different ways. First, the wind speed was interpolated to the nominal altitudes of the MASS layers (0.5, 1, 2, ... km) from the nearest NCEP layers. Alternavively, the wind speed from NCEP was averaged within the nominal width of the MASS layers, e.g. from

6 km to 12 km for a 8-km layer. Then the atmospheric time constants $\tau_{0,av}$ and $\tau_{0,int}$ were computed from (1) and compared with τ_{MASS}. This calculation of τ_0 automatically excludes turbulence below 0.5 km. The comparison plots (Fig. 5) are presented in grey-scale (as a density of points) in order to avoid cluttering.

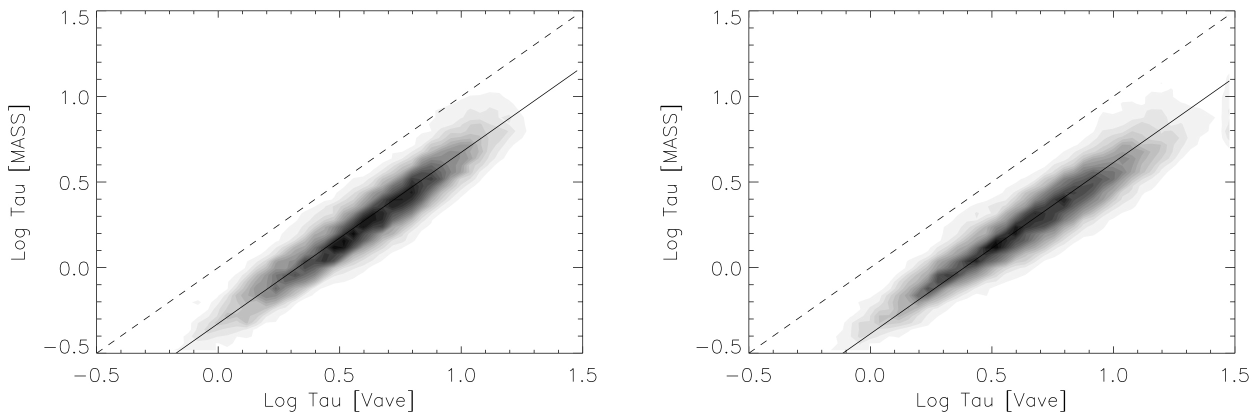


Figure 5: Comparison of the time constants measured by MASS (vertical axis) with the time constants calculated from the MASS turbulence profiles and wind speeds (left – averaged within MASS "layers", right – interpolated to the layer nominal altitudes). The dashed lines show equality, the full lines – estimated MASS bias.

Both methods of the time constant calculation show a very similar bias of MASS, $\tau_{MASS}/\tau_{0,av} = 0.47$. Otherwise, the correlation between these estimates is as good as can be expected, given the approximations involved.

Marc Sarazin compared the time constant measured by MASS with the approximate estimate computed from the seeing (DIMM data) by the recipe of [3]. The plot of Fig. 6 is taken from the ESO web site[1]. The scatter of points in this plot is large. Concentrating on the values close to typical (3 ms), we see that DIMM "measures" ∼2.5 times larger time constants than MASS.

5 Summary and discussion

The data on the bias in the time constant measured by MASS are summarized in Table 1. As noted, the bias is evaluated in the logarithmic sense by computing the average of $r = \log_{10}(\tau_{MASS}/\tau_0)$ and the rms scatter σ_r. The Table gives the number of "points" N used in calculating the bias.

The two most reliable estimates of the MASS bias come from the first two lines. The rms scatter of the bias from profiles is even less than from the simulations, probably because the range of the wind velocities contributing to the real τ_0 estimates is rather restricted. It is encouraging that both methods require the same adjustment of the calibration coefficient, to $K = 0.22$ ms. A large part of this bias disappears when the contribution of the low layers (un-sensed by MASS) is accounted for, so the actual τ_{MASS} turn out to be essentially un-biased. The scatter of the τ_{MASS} is about 27% relative to the un-biased τ_0 and only 13% relative to $\tau_{0,h}$.

[1] http://www.eso.org/gen-fac/pubs/astclim/paranal/asm/mass/MASS-Paranal-2003/

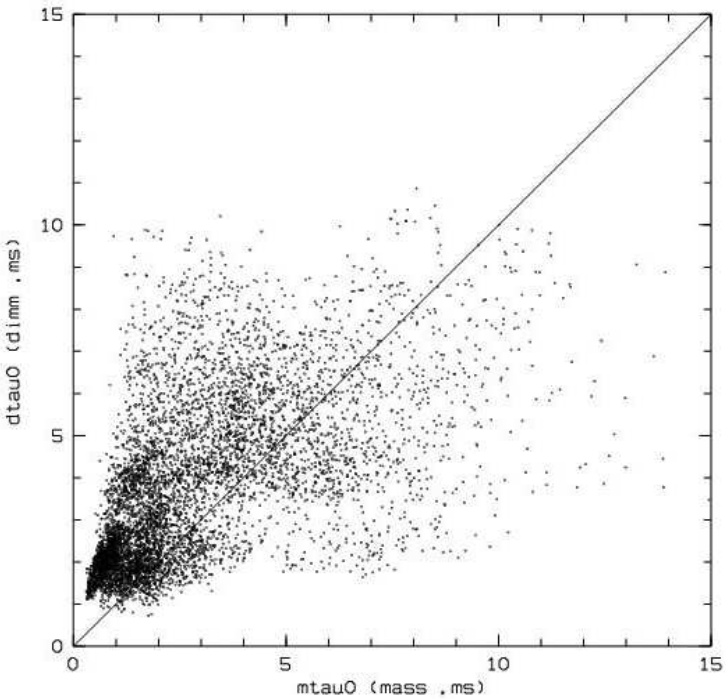


Figure 6: Comparison of the time constants measured by MASS (horizontal axis) with the time constants estimated from the DIMM (vertical axis) for Paranal. The diagonal corresponds to the equality.

Table 1: Estimates of the τ_{MASS} bias

Num.	Comparison data	N	$\langle r \rangle$	σ_r
1	Phase screens	19	-0.103	0.090
2	Pachón profiles, $h > 1$ km	23	-0.106	0.054
3	Pachón profiles, full	26	-0.017	0.102
4	Meteo data, average V	64504	-0.326	0.133
5	Meteo data, interp. V	64504	-0.387	0.143
6	DIMM + meteo	?	~ -0.4	-

The comparison of τ_{MASS} with the indirect τ_0 estimates derived from meteo-data hints that τ_{MASS} under-estimates τ_0 by as much as 2–2.5 times. Such a large bias cannot be explained by the intrinsic defects of the MASS method (an over-estimate is expected instead). Hence the comparison values are suspect.

The NCEP wind velocities were averaged without C_n^2 weighting and we may guess that layers with stronger turbulence move faster than the bulk of the atmosphere. This explanation implies a difference of ~ 1.6 times between the simple average V and the C_n^2-weighted average. However, the balloon profiles show the ratio of C_n^2-weighted to simple average wind speed (in 1-km bins) to be close to one (within 10%). Averaging witnin wide MASS "layers" does reduce the apparent wind speed. Figure 7 compares the median wind speed derived from the balloon profiles with the median speeds used to compute $\tau_{0,av}$. The 8-km layer shows the bias caused by the altitude averaging.

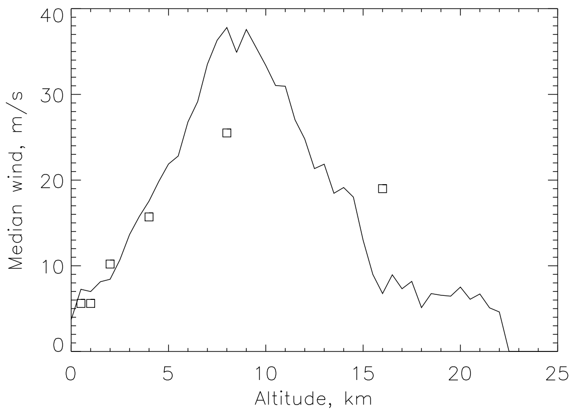


Figure 7: Median wind speed at Cerro Pachón vs. altitude computed from the 26 balloon profiles (line) and from the meteo data averaged within MASS layers (points).

6 Conclusions

It is found that the real MASS instrument measures the atmospheric time constant produced by high-altitude layers 0.79 times smaller than the true τ_0. This bias is, on the average, compensated by the un-measured contribution from the low layers, at least at Cerro Pachón. Other sites may have quite different fraction of the ground-layer (GL) turbullence, hence it is better to consider this contribution explicitly. The turbulence integral in the ground layer $(C_n^2 \mathrm{d}h)_{GL}$ is measured by the MASS-DIMM, and the GL wind speed is known from a local meteo-station. Hence, the un-biased time constant can be estimated as

$$\tau_0^{-5/3} = (1.27\,\tau_{MASS})^{-5/3} + (0.057)^{-5/3}\lambda_0^{-2}\;V_{GL}^{5/3}\;(C_n^2 \mathrm{d}h)_{GL}. \qquad (5)$$

The intrinsic accuracy of such estimate is expected to be ±20% or better.

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