Calculation of the Strehl ratio from 1-dimensional data

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1 Motivation

The DIMM instruments for measuring atmospheric seeing are sensitive to the optical quality of the stellar images formed by the individual sub-apertures. When these images are not perfect (aberrated), the seeing measurements become biased (over-estimated). To control this bias, we evaluate the Strehl ratios (SRs) from the real images. The SR values above 0.3 (or some higher threshold) are acceptable, a lower SR means that the DIMM data are biased.

Some DIMM instruments work in the “drift scan” mode, when the image is compressed in one dimension. How can we evaluate the Strehls from such 1D data?

2 Methods of Strehl estimation

Let $I(x, y)$ be the 2-dimensional intensity distribution in the image. We assume that the image is well sampled, i.e. the pixel size $p < \lambda/(2D)$, where λ is the wavelength and D is the aperture diameter. The pixel size p is in radians. Then the Strehl ratio S can be readily estimated from the formula

$$S_{2D} = \frac{1}{K} \frac{I_{max}}{\sum I}, \quad K = \frac{\pi}{4} \left(\frac{Dp}{\lambda}\right)^2. \tag{1}$$

The averaging over pixel is neglected here.

In case of one-dimensional (drift scan) data, we have access only to the compressed profile $J(x)$,

$$J(x) = \int I(x, y) \mathrm{d}y. \tag{2}$$

How can we recover the SR from such a profile? It is already clear that the answer can be only approximate. For example, an astigmatic image can be well focused in one dimension and will give a diffraction-limited profile $J(x)$, while its true SR will be less than 1.[1].

One idea, suggested by Matthias Shöck, is to correct the measured maximum intensity of the scan J_{max}, dividing it by the FWHM of the scan, L. It is hoped that $J_{max}/L \approx I_{max}$. Hence the SR can be estimated as

$$S_{1D} \approx \frac{1}{LK} \frac{J_{max}}{\sum J}. \tag{3}$$

[1] In this case the DIMM measurements are not biased, so the image quality can be considered perfect anyway

Another, alternative idea is to model a 2D image as a product of two identical functions, $I(x,y) = f(x)f(y)$, like e.g. a diffraction image formed by a square aperture. In this special case, $I_{max}/\sum I = (J_{max}/\sum J)^2$. Hence we can evaluate the SR as

$$S_{1Da} \approx \frac{1}{K} \left(\frac{J_{max}}{\sum J}\right)^2. \quad (4)$$

How good are the approximations (3) and (4) when applied to real images? To answer this question, I did some modeling.

3 Modeling

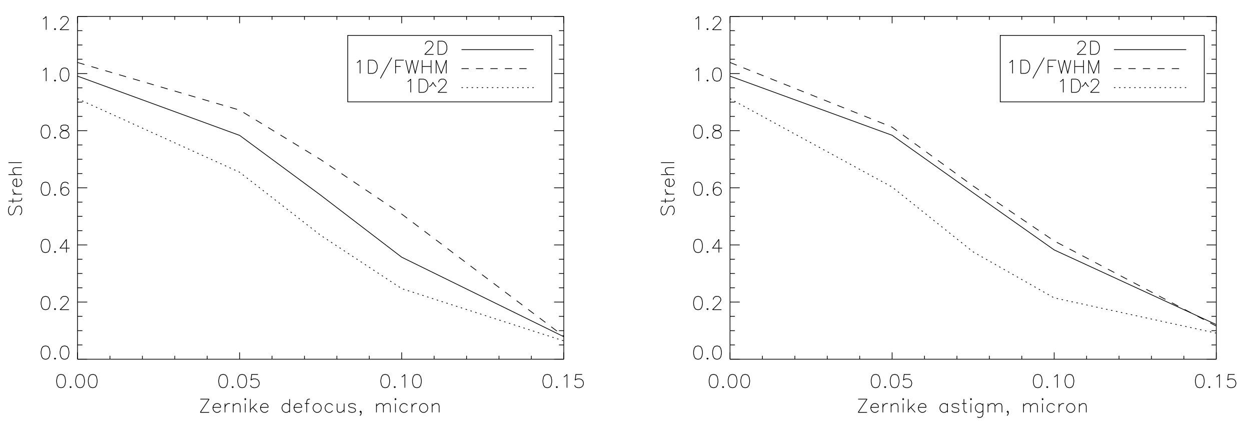

Figure 1: Strehl ratio calculated in different ways from defocused (left) or astigmatic (right) images. The horisontal axis shows the rms wave-front aberration in microns. The curve 1D/FWHM is Eq. (3), and the curve 1Dˆ2 – Eq. (4).

I calculated monochromatic images distorted by a variable amount of a low-order aberration – defocus or astigmatism. The SRs were estimated by the full 2D method (1) and by two alternative recipes (3) and (4). The calculations were done by the IDL code `DIMMstrehl.pro` with the parameters $D = 0.1$ m, $\lambda = 0.65$ μm, $p = 0.335''$, on a grid of 128^2 pixels.

The results (Fig. 1) show that neither of the two techniques gives a perfect SR estimate. The FWHM method (3) over-estimates the SR (even stronger if, instead of a real FWHM L, we divide by the diffraction-limited FWHM L_0). The squares method (4) under-estimates the SR. In case of a defocus, the average of the two estimates is quite close to the true SR.

It is possible to improve the estimates by introducing some coefficient, derived from modeling. For example, we can reduce (3) by some 4% or increase (4) by 9% in order to get SR=1 for a perfect image. Such adjustment is recommended, it will improve the accuracy of either method. Even then, the squares SR (4) will be pessimistic, and the FWHM SR (3) – optimistic. Either of these estimates (or both) can be used for controlling the optical quality of the spots in DIMM.