Checking the LuSci profile restoration

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1 Problems with the current restoration method

Lunar scintillometer, LuSci, serves for measurement of surface-layer turbulence at a number of established and new sites. An original 4-channel prototype worked at CTIO and at LCO, while 6-channel instruments are being used by ESO. Methods of extracting turbulence profile (TP) $C_n^2(h)$ from scintillation covariances are still a subject of research. Of interest are the accuracy and robustness of restored TPs.

The linear method of "layers" [2] was replaced in 2008 by a more elaborate model-fitting, representing the TP by linear (in log-log coordinates) segments between selected *pivot points* [4]. This technique is inspired by data analysis of SHABAR [1]. Meanwhile, the scintillometer array developed by the University of Vancouver fits data with double-exponential model [5].

It was demonstrated that the *pivot-point method* (PPM) produces results not very different from the previous *layers* method when applied to the 4-channel prototype [4]. However, the TPs derived from the 6-element LuScis systematically show low C_n^2 values at the 16-m point, which is un-realistic. Limited comparison of LuSci with SL-SLODAR at Paranal in October 2008 also demonstrated this effect. The reliability of the PPM is thus put in question, warranting further study.

2 Input data

Data from the ESO LuSci-1 instrument at Paranal on the nights of January 8,9,11 2009 was used to test the restoration. For the first 2 nights, the data were filtered by A.Berdja to remove a small fraction of faulty measurements, for Jan. 11 the data are not yet filtered.

The covariances are written in the `.dat` file in the following order: variances for 6 channels, covariances of ch.0 with channels 1-5, covariances of ch.2 with chs. 2-5, etc. Figure 1 plots the covariances averaged for the whole night in the same order as redorded in `.dat`. The covariances between ch. 0 and other channels (points from 6 to 11) correspond to baselines 19, 23, 25, 28, 40 cm and should decrease. This is true, except that point 6 is high. Covariances plotted vs. baseline do not decrease, as expected, but show some maxima. Initially, I suspected that the order of covariances is wrong.

The covariances were then re-calculated from the binary data using the `/RECOMP` key in `allproc`. The results, also plotted in Fig. 1, behave as expected. We looked into the data-acquisition code `lusci6.c` (E.Bustos, version of August 15, 2008) and have not detected any obvious errors. We note from Fig. 1 that the values of covariances, not just their order, are different.

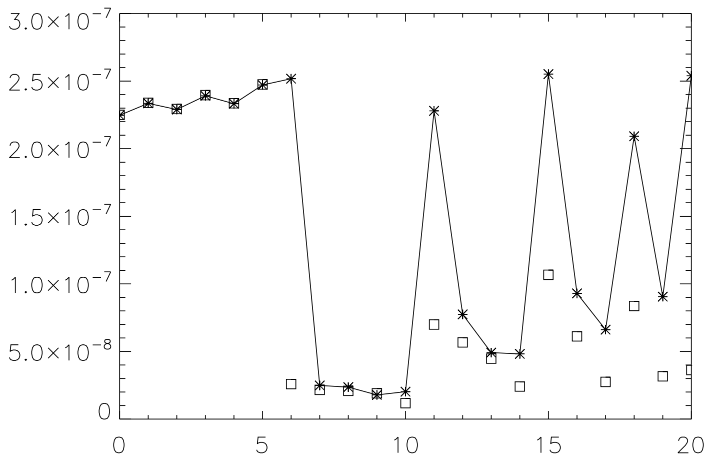

Figure 1: Mean covariances for the night of January 9, 2009, as recorder in the `.dat` file (line and asterisks) and re-computed from the `.bin` file (squares). [covmean.ps]

Conclusion 1. The covariances in the `.dat` files for 6-channel instruments seem to be corrupted by some un-identified error, they are wrong. The error must be fixed (there was no such error in the 4-channel code). Meanwhile, for the existing data we have to compute the covariances from the binary files.

Photon noise. The variance (zero baseline) contains some contribution from the amplifier and photon noise. If this noise, unaccounted for presently, is significant, it can distort the result and would be interpreted as excess of low-altitude turbulence.

It was shown that in normal operating conditions (bright Moon) the photon noise exceeds the amplifier noise. Scintillation index produced by photon noise is $s^2_{phot} = 1/(N\tau)$, where N is the flux, photons/s, and τ is the integration time or inverse of equivalent bandwidth. The average voltage in the AC channel of LuSci is $V = KRI$, where $K = 2$ is the amplification coefficient of the DC component at the 2-nd stage in ESO LuSci, $R = 10^7\,$Ohm is the load resistor, $I = q_e N$ is the photo-current, $q_e = 1.602\,10^{-19}\,$K is the charge of the electron. Combining these expressions, we find that

$$s^2_{phot} = \frac{KRq_e}{V\tau} = \frac{A}{V} = 1.6\,10^{-19}/V. \tag{1}$$

In typical conditions $V > 1\,$V, while the variance due to scintillation reaches 10^{-7}, therefore the contribution of the photon noise to the variance is $\sim 1\%$. A full noise model would include the term B/V^2 due to the amplifier noise. The noise parameters $[A, B]$ should be included in the par-file, specific for each instrument.

3 Tuning the restoration algorithm

Even with correct covariances, the results of PPM delivered by `profrest.pro` show obvious artifacts such as very low values of C_n^2 at some points. The reason is two-fold. First, the C_n^2 values calculated by a linear technique as initial approximations to the non-linear fitting are very noisy, often negative. The negative values are set to 10^{-19}, but the non-linear fitting algorithm, `AMOEBA`, does not always recover the true solution starting from these very erroneous starting values. This problem was fixed by using previous TP as starting point for fitting. Only the first measurement is treated by the linear method for the initial guess. Moreover, the parameters $y = \log(C_n^2)$ determined by the linear method are smoothed before starting the fitting.

The second reason for unstable results is that the inverse problem is intrinsically ill-conditioned, so small fluctuations of the initial data can lead to large errors in the TPs. The total turbulence intensity (hence seeing) is well constrained by the input data, but it is attributed to different altitudes, depending on the noise. The TP displays vs. time often shows "flips" between pivot points. To fix this, we add a *smoothness penalty* to the function being minimized,

$$\chi^2 = (1/N) \sum_{i=0}^{N-1} [(B_i - B_{i,mod})/B_0]^2 + \alpha S \tag{2}$$

where

$$S = \sum_{k=1}^{K-1} |y_k - 0.5(y_{k-1} + y_{k+1})|. \tag{3}$$

The goodness-of-fit parameter χ^2 is the average distance between measured covariances B_i and the covariances corresponding to the TP model, $B_{i,mod}$, where $i = 0, ...N-1$ is the baseline index including zero baseline (variance), $N = 16$ baselines for a 6-channel instrument. The difference is normalized by the variance B_0, so $\sigma = \sqrt{\chi^2}$ is a convenient measure of the fitting error. Typically, we reach $\sigma \sim 0.02$, $\chi^2 \sim 4 \cdot 10^{-4}$. Analysis of statistical errors of the input data B_i [3] shows that their errors are always of comparable magnitude, therefore we do not weight the residuals in (2) by errors and keep it simple.

The measure of TP smoothness S is the sum of 2-nd differences over K pivot points, in logarithmic sense. It is added to χ^2 with a regularization coefficient $\alpha = 10^{-4}$. If the restored TP has a "spike" of 1 dex, the typical χ^2 will increase by 25%. Regularization helps to select among many solutions compatible with the data the smoothest one. We tried different values of the regularization parameter α and have chosen the smallest value that still has some effect on the result. Any linear distribution of y corresponding to a power-law TP has $S = 0$, no penalty.

Regularization helps to stabilize the solution. We can then increase the number of the pivot points from 4 to 5, after noting that covariances at smallest baselines were showing some systematic residuals, calling for a better model at low altitudes. The pivot points' distances are fixed in the code at [3, 12, 48, 192, 768] m. Now we do not scale these distances with airmass, keeping them at the same range. This also helps to model low layers better, while the TP model is still useful for computing $C_n^2(h)$ for any given h. On the other hand, the extra parameters `zint`, altitudes to which turbulence integrals are calculated from the TP model, are still defined above instrument, not along the line of sight. The number of these user-defined altitudes can be large (up to 20).

4 Examples

The new restoration code `profrest3.pro` implements the tuned algorithm. The input data are covariances produced by `allproc`. The format of the output files is changed. Now the columns of `.tp` file contain: Julian date, air mass, GL seeing, fitting error σ, 5 values of $y = \log_{10} C_n^2$ at pivot points, and the user-defined number of values of turbulence integrals from the intrunment to the altitudes `zint`, in $\mathrm{m}^{1/3}$.

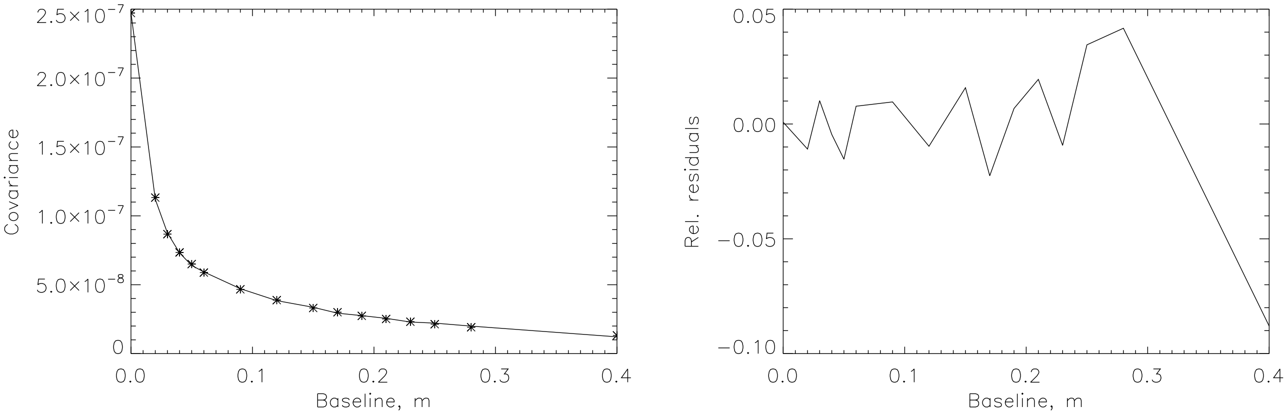

Figure 2: Left: average covariance vs. baseline for January 9, 2009 (line) and average modeled covariance (points). Right: relative residuals between average covariance and its model for the same night. [covmod.ps, covres.ps]

Figure 2 shows the dependence of average covariances (Jan. 9, 2009, LuSci-1 data) on the baseline, a smoothly declining curve. The points correspond to the average model, showing the absence of systematic errors. On the right-hand panel, the residuals normalized to B_i (not to B_0) are plotted. The last point at 40-cm baseline shows the largest systematic deviations, simply because the covariance is the smallest.

Figure 3 contains the plots of TP and GL seeing generated from the `.tp` file with `xmgrace` scripts `plotcn2.awk` and `plotsee.awk`, respectively. The $\log C_n^2$ values at pivot points are plotted (the actual altitudes of these points depend on the air mass). We see that the TP generally (but not always) decreases with altitude. In the second half of the night, the surface layer was very strong and very thin. Data for another night are plotted in a similar way in Fig. 4. Here the TP is almost constant in the first 50 m above ground, unlike the other night, while the GL seeing is simular. LuSci thus helps to measure the thickness of the ground-layer turbulence and to extrapolate the seeing neasured by a site monitor to the height of telescdope domes.

Figure 5 shows data for yet another night, January 11, 2009. The data were of good quality, but not filtered, hence some spikes are present. The point here is to show that the restoration algorithm which uses wrong "spiky" values as starting point for fitting the next data recovers the same values as before, demonstrating good stability.

The C_n^2 values at the last pivot point show large, often iregular variations. These data are not reliable, as they are poorly constrained by the covariances. The impact of the last pivot point on the reconstructed GL seeing is seen to be negligibly small. This point is important, tough, for fitting the data, as it permits to account for high-altitude turbulence and, sometimes, for additional errors due to transparency variations [4].

Conclusion 2. The tuned restoration algorithm now produces TPs as expected, with $C_n^2(h)$

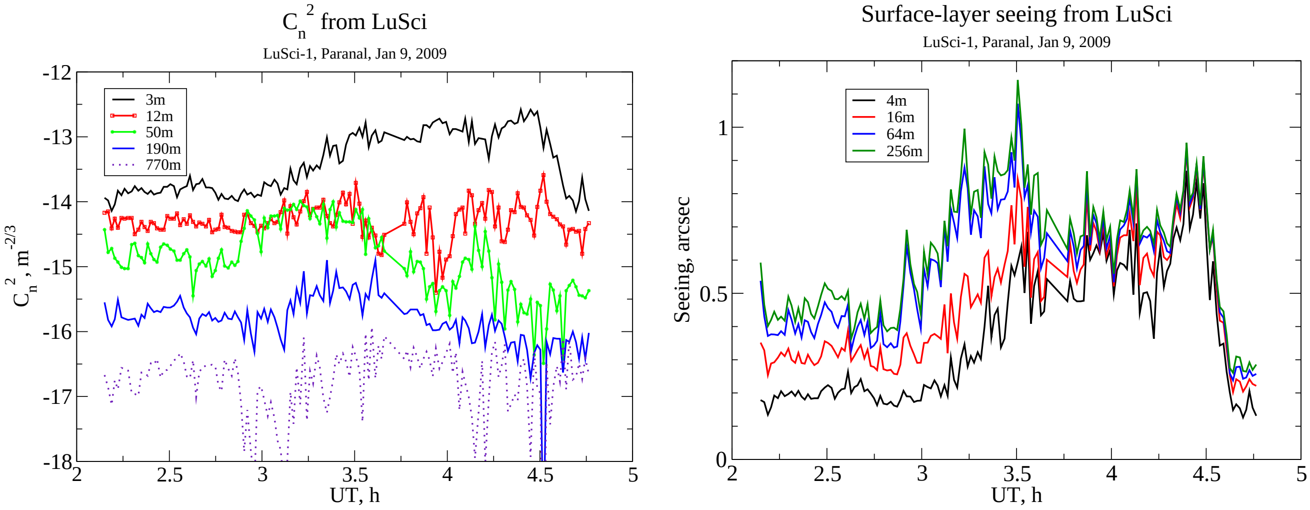

Figure 3: Turbulence profiles (left) and seeing integrated up to selected altiitudes (right) for the night of January 9, 2009. [090109tp.eps, 090109see.eps]

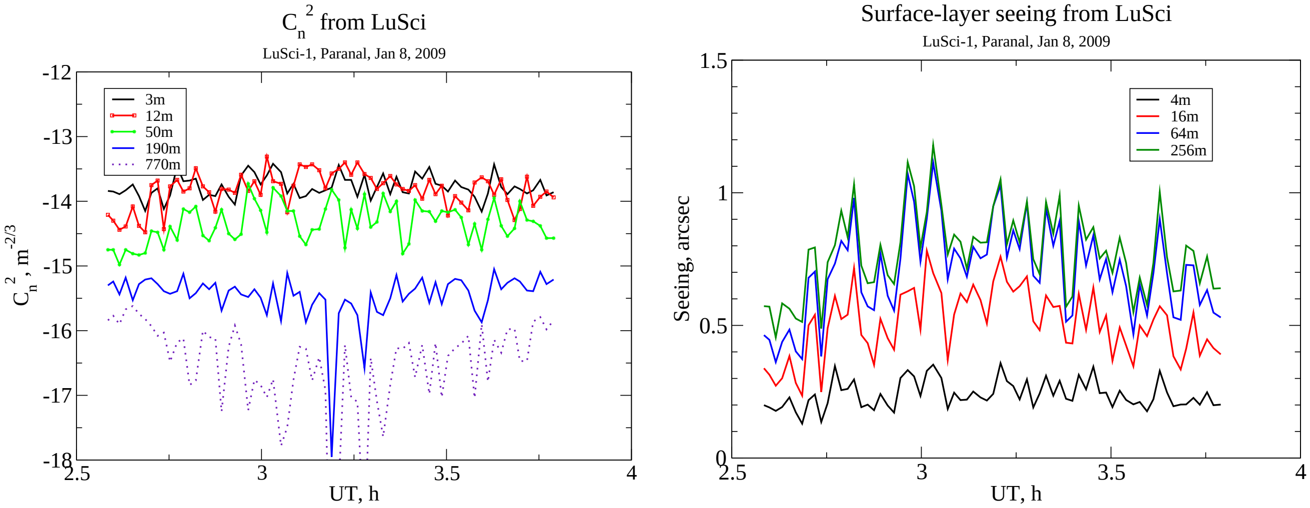

Figure 4: Turbulence profiles (left) and seeing integrated up to selected altiitudes (right) for the night of January 8, 2009. [090108tp.eps, 090108see.eps]

typically decreasing with height. Input covariances are fitted to within few percent. A higher fitting accuracy is not warranted by the approximations used so far (neglect structure of the Moon image, thick turbulent layers, etc.).

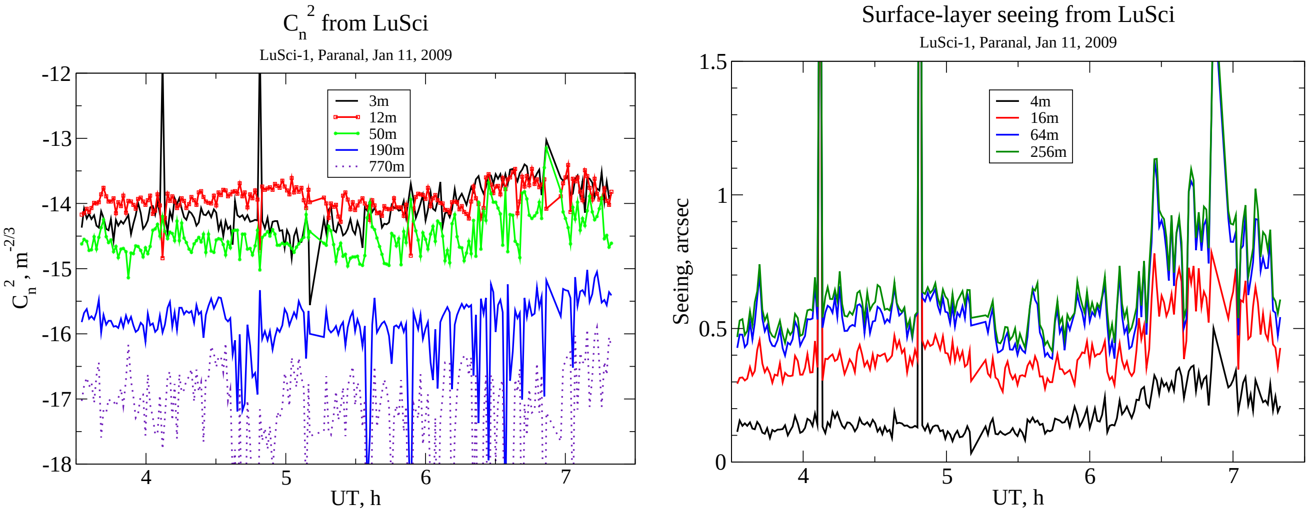

Figure 5: Turbulence profiles (left) and seeing integrated up to selected altiitudes (right) for the night of January 11, 2009. [090111tp.eps, 090111see.eps]

5 Further work

Data filtering should be incorporated in the `datproc`, to produce "clean" covariances in single operation, while documenting the data quality at the same time.

Existing data must be re-processed and compared to SL-SLODAR and MASS-DIMM.

References

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[2] Tokovinin A. Restoration of turbulence profile from lunar scintillation. January 22, 2007. http://www.ctio.noao.edu/˜atokovin/profiler/restoration.pdf

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[5] Hickson, P., Pfrommer Th., Crotts A.P. Optical turbulence profiles at CTIO from a 12-element lunar scintillometer. Preprint, 2009.