Diffraction effects in MASS: generalized mode

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1 Introduction

MASS instrument measures scintillation indices (normal and differential) scintillation indices in 4 concentric apertures. The theory is well developed for the normal mode, when the apertures are conjugated to the pupil plane. However, in order to sense the near-ground turbulence, we need to conjugate the apertures to some negative range z_0 (0.5 or 1 km). A virtual propagation path is added, allowing for the lower layers to develop some scintillation. The theory remains valid, but *only for infinite pupil* of the entrance telescope. We hope to use MASS with a small 14-cm feeding telescope. Upon propagation over the virtual path, diffraction on this entrance pupil (not to be mixed with MASS apertures!) distorts the intensity distribution, hence may influence the measured scintillation indices.

It was presumed initially that the central part of the beam (where the two smallest MASS apertures are located) will be free from diffraction, fulfilling the infinite aperture approximation. This is not the case, however: even after propagation over 500 m there is no "clear space" in the 14 cm beam, intensity is modulated by few tens of percent, compromising the principle of "generalized" mode. Moreover, in a real instrument the pointing will experience some jitter α, hence the MASS apertures will move with respect to entrance aperture by αz_0. Even in the absence of turbulence, motion of the MASS apertures over diffracted image of the pupil should cause some modulation of light.

In this study, we address these issues by means of numerical simulations. A pair of 2 cm and 4 cm concentric apertures is considered (representative of the two smallest MASS apertures) and scintillation indices are computed for a large number of simulated phase screens with known properties. The results are compared to the theoretical predictions for infinite entrance aperture and it is shown that MASS will be operational in the generalized mode. Some theoretical interpretation of these results is provided.

2 Simulation tools

2.1 Turbulence simulator

Simulation of turbulence is widely used nowadays, e.g. in adaptive optics. As a starting step, we took the "Turbulenz" code in IDL from Heidelberg (*http://www.mpia-hd.mpg.de/AO/ATMOSPHERE/TurbuLenZ/tlz.html*). Given the Fried parameter r_0 for a layer, the code outputs a phase screen generated by spectral technique: the Fourier Transform (FT) of the phase is generated as array of zero-mean Gaussian random numbers, their amplitudes increasing at low frequencies f as $f^{-11/6}$. The underlying phase power spectrum W_ϕ (in square meters) is

$$W_\phi = 0.023 f^{-11/3} r_0^{-5/3}. \quad (1)$$

It is well known that this method underestimates the low-frequency part of turbulence, and, notably, produces wrong phase structure functions. The reason is that any function obtained by discrete FT is periodic, with a period equal to the grid size. Thus, the structure function initially increases, reaches a maximum at half-grid-size and then gracefully drops to zero again at a separation equal to the grid size. Additional steps are taken to overcome this (adding sub-harmonics), but then the simulated phase is no longer periodic. We are interested here in the scintillation, which effectively filters only high spatial frequencies. So, no sub-harmonics were added.

2.2 Propagation

The propagation of wave-fronts is simulated by the spectral technique, again using the FT. A code in C for light propagation was written by Gleb Vdovin and can be found at *http://www.okotech.com/*. Briefly, if $U_1(\vec{r})$ is the amplitude of the light waves before propagation, and $A_1(\vec{f})$ is its FT, then the FT of amplitude after propagation is obtained by frequency filtering:

$$A_2(\vec{f}) = A_1(\vec{f}) \exp(-i\pi z \lambda f^2), \tag{2}$$

where z is the propagation distance, $f = |\vec{f}|$.

This method is computationally fast, involving only two FFTs. Its drawback, however, is that in fact it simulates propagation in a rectangular waveguide of grid size with reflective walls. In order to approximate propagation in the free space, numerical "absorption" is sometimes added near the "walls". Alternatively, beam size must be some 2 times smaller than the grid size, so that the intensity near the walls is negligible. This is the case of MASS simulation: for a 14-cm aperture, grid size of 30 cm is adequate.

Propagation of periodic phase screens does not present problems near the grid boundaries, because amplitude continues smoothly outside the boundaries. This is why in our simulations periodic phase screens can be propagated any distance without artifacts near grid boundaries.

2.3 Simulation code

The IDL simulation code `massim.pro` is listed in the Appendix. In Fig. 1 the geometry of the problem is illustrated. A phase screen corresponding to some turbulent layer is generated. Corresponding complex amplitude of the light wave is then propagated over a distance z to the entrance aperture of MASS (diameter $d = 14$ cm). After aperture cutoff, further propagation over a distance z_0 (virtual propagation path in generalized mode) is applied. The intensity distribution is found as square modulus of amplitude and integrated within the two full circular apertures (inner and outer diameters d_1 and d_2) to give the fluxes I_1 and I_2. Light flux in the outer annular aperture 2 is equal to $I_2 - I_1$.

After repeating the calculations N_{it} times (typically $N_{it} = 400$), the resulting arrays of fluxes are processed to compute the scintillation indices, computed as dispersions of natural logarithms. For example, the differential scintillation index s_d is equal to

$$s_d = \langle x^2 \rangle - \langle x \rangle^2, \quad x = \ln \frac{I_2 - I_1}{I_1}. \tag{3}$$

Expected theoretical indices are computed from the weighting functions $Q(z, \lambda)$ and turbulence integral J:

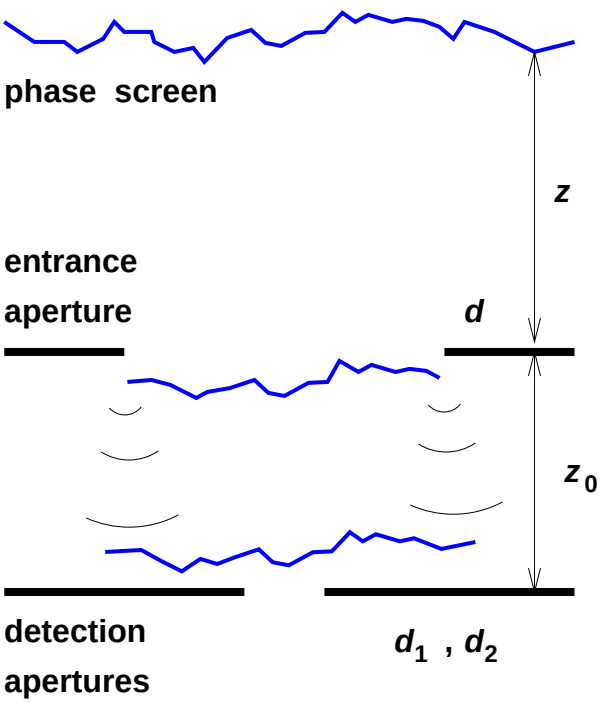

Figure 1: Geometry of the problem.

$$J = \int C_n^2(z)\mathrm{d}z = \frac{1}{0.423\ r_0^{5/3}} \left(\frac{\lambda}{2\pi}\right)^2. \quad (4)$$

$$s_d = J\, Q_d(z + z_0, \lambda). \quad (5)$$

We verified that the indices given by Monte-Carlo simulations are equal to the theoretical values within statistical uncertainties. This holds, of course, only for weak scintillations, $s_1 < 0.3$, when Rytov approximation remains valid. Thus, our phase-screen and propagation codes are validated.

To simulate pointing jitter, additional linear phase term is added at the entrance aperture. The rms dispersion of jitter α is given as input parameter. Thus, two Gaussian normal variables α_x and α_y with dispersion α_0^2 are generated and the amplitude at the entrance is multiplied by

$$U_{tilt} = \exp\left[\frac{2\pi i}{\lambda}(\alpha_x x + \alpha_y y)\right]. \quad (6)$$

3 Results

In Fig. 2 the central cut of an intensity distribution for an ideal diffraction pattern is displayed. Aperture diameter is 14 cm, propagation distance 1 km, wavelength 0.5 micron. The severity of Fresnel diffraction is evident.

In Fig. 3 we show four realizations of the intensity pattern that is likely to be observed in the plane of MASS apertures. Here, a single turbulent layer with $r_0 = 0.2$ m at a distance of 1 km from the instrument is simulated, with additional 1 km of propagation on the virtual path. We added a severe

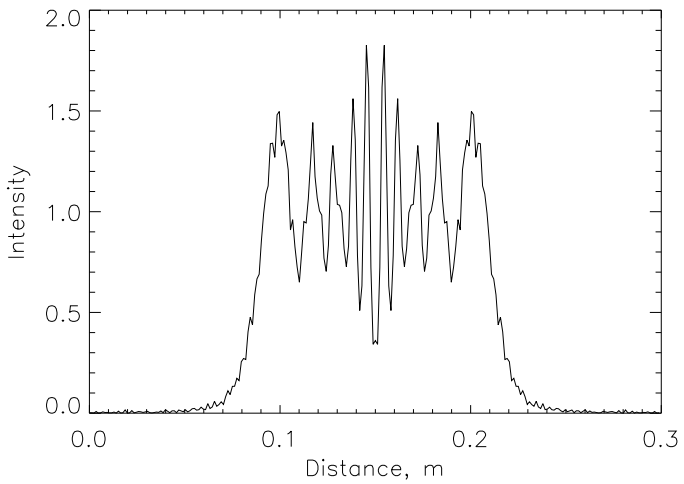

Figure 2: The cut of an intensity distribution for a 14 cm entrance aperture after propagation for 1 km.

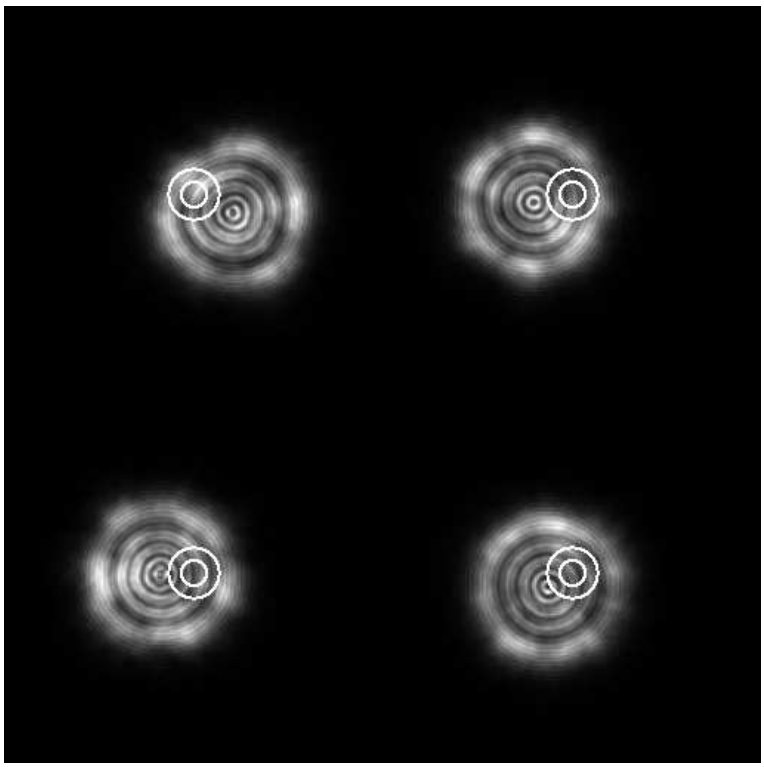

Figure 3: Four independent realizations of intensity distribution at the double apertures, marked by circles. See the text.

pointing jitter of rms amplitude $\alpha = 4''$. Thus, MASS apertures of 2 and 4 cm diameter are displaced randomly from the center of the beam cut by the entrance aperture. Intensity distribution is a result

of interplay between turbulence and diffraction at the entrance aperture.

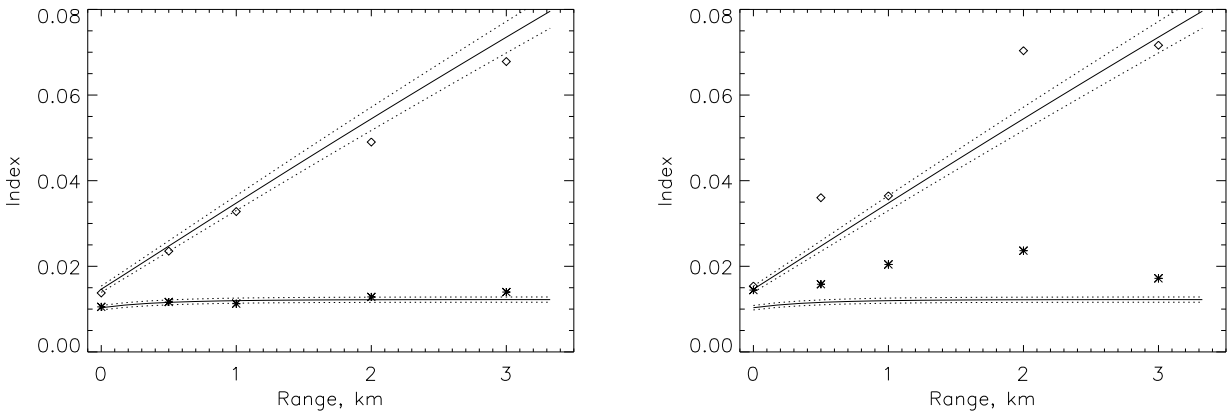

Figure 4: Expected scintillation indices are plotted in full lines for the inner 2 cm aperture and for differential $2+4$ cm aperture as a function of layer range z (for fixed virtual propagation $z_0 = 1$ km). Diamonds and asterisks show the Monte-Carlo results for $N_{it} = 400$, dotted lines – 1σ deviation from the theoretical curves due to finite sample size. Left: jitter $1''$, right – jitter $4''$.

The comparison between theoretical and simulated scintillation indices is shown in Fig. 4. We omitted for clarity the indices in the large 4 cm aperture, which are intermediate between 2 cm normal indices and differential indices. The figure shows that, despite diffraction, the measured indices correspond to the theoretical ones for a moderate pointing jitter. The jitter of $4''$ is already too big, and we see that the simulated indices are systematically higher than expected. For a $3''$ jitter the results are still good.

We conclude that MASS can work in the generalized mode and will give correct results when theoretical weighting functions are used for data interpretation. More specifically, seeing can be estimated from the differential index with two smallest apertures. It was shown previously that the virtual propagation can be reduced to 0.5 km. Thus, our tests represent the worst case (largest virtual path, small aperture). It appears that this result is robust. We varied number of points in the grid, grid size, r_0 and obtained reasonable agreement between Monte-Carlo and theoretical indices.

4 Why does it work?

Remarkable immunity of MASS to the diffraction at the entrance aperture calls for explanation. For small perturbations, intensity fluctuations are proportional to the fluctuations of the real part of complex amplitude. Thus, intensity integrated over MASS aperture is determined by the real part of the weighted sum of complex amplitudes of the light waves that enter the instrument. The weights are, of course, complex and are determined by a combination of propagation and cutoffs within MASS (Fig. 1). However complicated, it is only a spatial filter applied to the random light field at the entrance. If we prove that this filter is not influenced by the addition of the entrance aperture, the phenomenon will be understood.

The complex filter acting at the input light field can be obtained by "back propagation". We suppose that the MASS apertures are illuminated from below by a parallel axial beam and compute

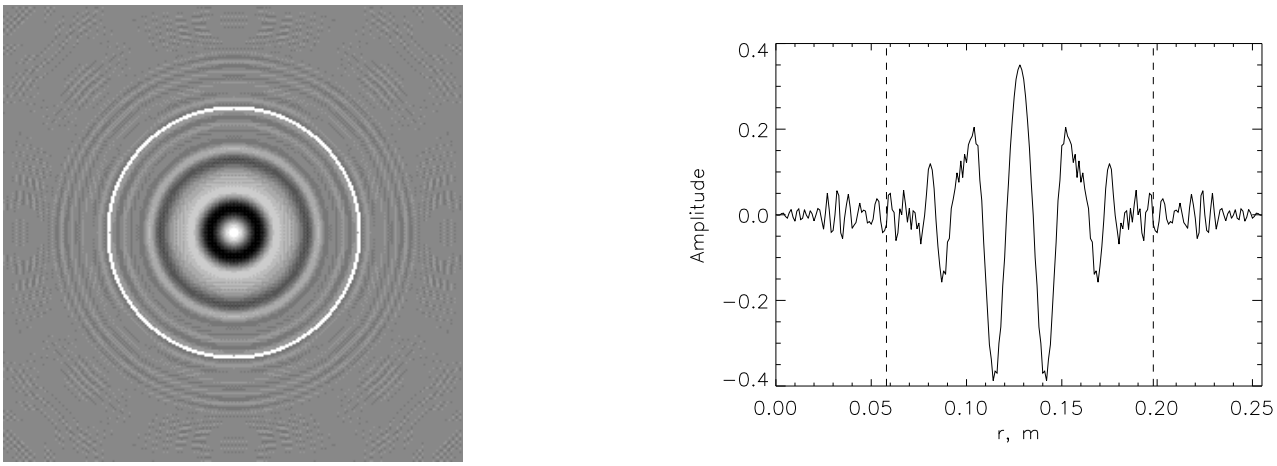

Figure 5: Distribution of the real part of the complex amplitude after back-propagation from double aperture to the entrance aperture. Left – two-dimensional, right – radial cut. Entrance aperture of 14 cm is depicted by the circle and by the dashed lines, respectively. The "ripple" is an artifact of the propagation code (reflections from walls).

the distribution of the complex light amplitude at the plane of the entrance aperture. The same propagation tool is used, of course. When the MASS aperture becomes very small, point-like, we obtain the classical case of Fresnel diffraction and our filter resembles a zone plate. Increasing the aperture leads to convolution which damps the fast oscillations in the outer parts of this zone plate. In case of differential scintillations, out spatial filter has a zero average, so the double MASS aperture must be represented by a positive (inside) and negative (outside) parts, with a total integral of zero.

In Fig. 5 we show the real part of the complex filter that corresponds to the differential combination of 2+4 cm apertures and 1 km virtual propagation. As already known, this filter isolates some intermediate spatial frequencies.

Now, the difference between the "pure" propagation in free space and the propagation after entrance aperture can be evaluated as a difference between the full filter and its central part cut by the entrance aperture. We see that in our case the part of the filter outside 14-cm boundaries is small and, moreover, high-frequency. Removing this part does not change appreciably the reaction of this filter to the atmospheric distortions, dominated by low spatial frequencies. This explains why MASS will work in the generalized mode.

For non-differential indices, the complex filter has a non-zero average. It is dominated by the central maximum, which becomes more prominent and wide as we enlarge the MASS aperture. Finally, when it reaches 14 cm, the wings spill outside the entrance cutoff, and the scintillation indices will be affected. Fortunately, we do not need to work with large apertures in the generalized mode.

Adding random jitter slightly complicates the things, introducing relative shifts between the filter and its cutoff. Still, we see that in our particular case shifts as large as 2 cm are tolerable. This corresponds to $4''$ at a distance of $z_0 = 1$ km. Pointing jitter of rms amplitude $2'' - 3''$ should be tolerable, as confirmed by simulations.

A Simulation code massim.pro

```
; Monte-Carlo simulation of MASS operation
; Only atmospheric propagation is considered, no noise
; Nov 12, 2001
; needs weight.pro to compute theoretical weighting function
;-------------INPUT PARAMETERS: ------------------------
; NGRID - half-size of simulation grid, pixels (64 recommended)
; SIZE - physical size of grid, meters (0.3 recommended)
; NITER - number of random phase screens (400 recommended)
; LAMBDA - wavelength, m
; R0 - Fried parameter of the layer, m
; D - telescope aperture diameter, m (0.14)
; D1 - inner aperture diameter, m (0.02 or 0)
; D2 - outer aperture diameter, m (0.04)
; Z0 - virtual propagation distance, m (1000)
; Z - distance to the turbulent layer, m
; JITTER - rms ampl. of telescope tracking jitter, arcsec (if <0 - atmosph.)
;--------------OUTPUT PARAMETERS ----------------------
; array S[6]: differential, 1st, 2nd indices (simul.), same theoretical
;-------------------------------------------------------
function massim, ngrid,size,niter,lambda,r0,d,d1,d2,z0,z,jitter
;
fact = sqrt(0.023)*(size/r0)^(5./6.)  ; for phase simulation
;
if (jitter lt 0.) then $
tilt_fact = 2.591*r0^(-5./6.)*d^(-1./6.)*size/(2*ngrid) $ ; atm. tilt factor:
else  tilt_fact = (jitter/206265.)*2.*!pi/lambda*size/(2*ngrid)

x_vec = findgen(2*ngrid)-ngrid     ; for tilt simulation
one_vec = replicate(1.0, 2*ngrid)  ; vector with value '1'

r = shift(dist(2*ngrid),ngrid,ngrid) ; radial distance from grid center, pixels
r[ngrid,ngrid] = 1.e-3
ur=r*0. ; array of zero value
in_d1 = where(r le (d1/size)*ngrid)
in_d2 = where(r le (d2/size)*ngrid)

int1 = fltarr(niter) ; light intensity in the 1st aperture
int2 = fltarr(niter) ; light intensity in the 2nd aperture

; circular telescope aperture U0
in = where(r le (d/size)*ngrid)
ur(in)=1.    ; input amplitude: real part
u0 = complex(ur, ur*0.)
```

```
; the Fresnel filters for two propagations: Z0, Z
farg = !pi*z0*lambda/(size^2)*r^2
fresnel = complex(cos(farg), sin(farg))
farg = !pi*z*lambda/(size^2)*r^2
fresnel2 = complex(cos(farg), sin(farg))

;---------  Main loop ----------------------
print, 'Computing ',niter, ' iterations...'
count=0
for i=0,niter-1 do begin

;--------- simulate turbulence--------------
ampl = fact*r^(-11./6.)*complex(randomn(s,2*ngrid,2*ngrid),randomn(s,2*ngrid,2*ngrid))
ampl[ngrid,ngrid] = complex(0.,0.)
phase = float(shift(fft(shift(ampl,ngrid,ngrid),/inverse),ngrid,ngrid))
u1=complex(cos(phase),sin(phase))      ; complex amplitude at turbulence layer

; ------ propagate wavefront to the pupil -----------------
if z gt 0. then begin
  a2 = fresnel2*shift(fft(shift(u1,ngrid,ngrid)), ngrid, ngrid)
  u1 = shift(fft(shift(a2,ngrid,ngrid),/inverse), ngrid, ngrid)
endif

; add tilt and aperture cutoff
tilt= randomn(s)*x_vec#one_vec+ randomn(s)*x_vec##one_vec
tilt = tilt_fact*tilt
u1=u1*u0*complex(cos(tilt),sin(tilt))

; --------- propagate to MASS apertures -----------------
a2 = fresnel*shift(fft(shift(u1,ngrid,ngrid)), ngrid, ngrid)
; amplitude after propagation
u1 = shift(fft(shift(a2,ngrid,ngrid),/inverse), ngrid, ngrid)
; intensity
int = abs(u1)^2

int1(i)= total(int(in_d1)) ; intensity in the 1st aperture
int2(i)= total(int(in_d2)) ; intensity in the 2nd aperture

; display the progress of calculation on screen
count=count+1
if count ge niter/10 then begin
     print, format='(a1,"-",$)', string(i*10/niter,format='(I1)')
     count=0
     tvscl, int   ; display 10 frames for the whole simulation
endif
;     tvscl, int   ; display all frames: makes nice movie!
```

```
endfor
; --------- end of main loop -----------------------------
print, ' Done!'

lint1 = alog(int1)
lint2 = alog(int2)
lint = alog((int2-int1)/int1)

scint = fltarr(6) ; to store results
scint[0] = (total(lint^2) - (total(lint))^2/niter)/(niter-1.)
scint[1] = (total(lint1^2) - (total(lint1))^2/niter)/(niter-1.)
scint[2] = (total(lint2^2) - (total(lint2))^2/niter)/(niter-1.)

; Compute the expected indices: needs weight.pro
turb_int = r0^(-5./3.)*(0.5*lambda/!pi)^2./0.423 ; Cn2 integral, m^(1/3)
scint[3] = turb_int*weight((z+z0)/1000.,lambda*1e6,d2*100., d1/d2)
scint[4] = turb_int*weight((z+z0)/1000.,lambda*1e6,d1*100., 0.)
scint[5] = turb_int*weight((z+z0)/1000.,lambda*1e6,d2*100., 0.)

return, scint
end
;-------------------------------------------------------
```