MASS response in strong scintillation

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1 The problem

The MASS instrument measures scintillation indices (normal and differential) in 4 concentric apertures to reconstruct low-resolution turbulence profile. The reconstruction is based on the weak-perturbation theory, where a relation between the turbulence intensity J and the scintillation index s is assumed to be linear, $s = WJ$. Here W is the weighting function which depends on the altitude of the layer, aperture geometry and wavelength.

In practice, conditions of strong scintillation are encountered where the index in the smallest 2-cm aperture exceeds 0.3 and can even reach 1. This happens when strong turbulent layers at high altitude are present. It is of interest to study the reaction of MASS to strong scintillation and to evaluate the bias that may result in applying the linear theory in this case.

In strong-scintillation regime, the method of index calculation becomes important. In theory, the fluctuations of the normal logarithm of the light intensity is the right quantity to compute. So, if x and y are the intensities in two apertures, the normal scintillation index s_x and the differential index s_{xy} should be computed as

$$s_x = \langle (\log x - \overline{\log x})^2 \rangle, \quad s_{xy} = \langle [\log(x/y) - \overline{\log(x/y)}\ ]^2 \rangle. \tag{1}$$

In fact, the calculations in the MASS software are done by different formulae that replace logarithms with ratios. These "linear" formulae are better suited for the subtraction of photon noise because the latter can be evaluated theoretically. The photon noise can be quite large, so the use of logarithmic formulae for index calculation seems problematic. The linear one are

$$s_x = \langle (x/\overline{x})^2 \rangle - 1, \quad s_{xy} = \langle (x/\overline{x} - y/\overline{y})^2 \rangle. \tag{2}$$

In this Report, I study the dependence of the scintillation indices calculated by both linear and logarithmic formulae on the strength of the scintillation and compare the results with the linear (non-saturated) scintillation theory.

2 Simulation tools

Given the Fried parameter r_0 for a layer, the code outputs a random phase screen generated by spectral technique: the Fourier Transform (FT) of the phase is generated as array of zero-mean Gaussian random numbers, their amplitudes increasing at low frequencies f as $f^{-11/6}$. It is well known that this method underestimates the low-frequency part of turbulence, and, notably, produces wrong phase structure functions. The reason is that any function obtained by discrete FT is periodic, with a period equal to the grid size. Thus, the structure function initially increases, reaches a maximum at half-grid-size and then gracefully drops to zero again at a separation equal to the grid size. Additional steps are usually taken to overcome this (adding sub-harmonics), but then the simulated phase is no longer periodic. We are interested here in the scintillation, which effectively filters only high spatial frequencies. So, no sub-harmonics were added.

The propagation of wave-fronts is simulated by the spectral technique, again using the FT. Briefly, if $U_1(\vec{r})$ is the amplitude of the light waves before propagation, and $A_1(\vec{f})$ is its FT, then the FT of amplitude after propagation is obtained by frequency filtering:

$$A_2(\vec{f}) = A_1(\vec{f}) \exp(-i\pi z \lambda f^2), \tag{3}$$

where z is the propagation distance, $f = |\vec{f}|$.

This method is computationally fast, involving only two FFTs. Its drawback, however, is that in fact it simulates propagation in a rectangular waveguide with reflective walls equal to the grid size. In order to approximate propagation in the free space, numerical "absorption" is sometimes added near the "walls". Alternatively, beam size must be some 2 times smaller than the grid size, so that the intensity near the walls is negligible. This is the case of MASS simulation: for a 14-cm aperture, grid size of 30 cm is adequate. Moreover, the propagation of periodic phase screens does not present problems near the grid boundaries, because amplitude continues smoothly outside the boundaries. This is why in our simulations periodic phase screens can be propagated any distance without artifacts near grid boundaries.

The IDL simulation code is `massim1.pro`.

3 Results and discussion

I simulate one turbulent layer at altitude 8 km with varying intensity corresponding to r_0 from 0.4 m to 0.04 m (at 500 nm wavelength), or seeing β from 0.25″ to 2.5″. Corresponding turbulence integrals $J = 6.8 \cdot 10^{-13} \beta^{-5/3}$ vary by two orders.

Light is monochromatic with wavelength of 500 nm. Two MASS apertures A and B have diameters of 2 and 4 cm, respectively, with B being annular. The simulation grid is 128^2 pixels and 0.32 m across. Thus, pixel size is about 2.5 mm and there is sufficient resolution to approximate the apertures. For each J, 1000 random wave-fronts are generated, so the expected statistical accuracy of the indices is around $1000^{-1/2} \sim 3\%$. Fig. 1 was generated by `nonlin.pro` and plots the results of these simulations.

The validity of the linear theory for weak scintillations ($s_A < 0.3$) is confirmed. As expected, there is no significant difference between logarithmic and linear formulae in this regime.

For strong (saturated) scintillations, $s_A > 0.3$, the indices saturate while the linear theory predicts their unlimited growth. The logarithmic formula saturates faster, so the linear formula (actually used by MASS) is preferable.

Scintillation produced by a high layer at altitude z at has a typical correlation length equal to the Fresnel radius $\sqrt{\lambda z} \approx 6$ cm. Thus, fluctuations in the apertures A and B are strongly correlated, while the intensity ratio A/B is not very different from 1, even in the saturated regime. This explains why there is no difference between linear and logarithmic formulae for the differential index s_{AB} and why this index matches well the theory even in this regime.

Conclusions. This study shows that under strong scintillation the measured indices in MASS also saturate. This should lead to under-estimation of the turbulence strength. Yet, quite opposite

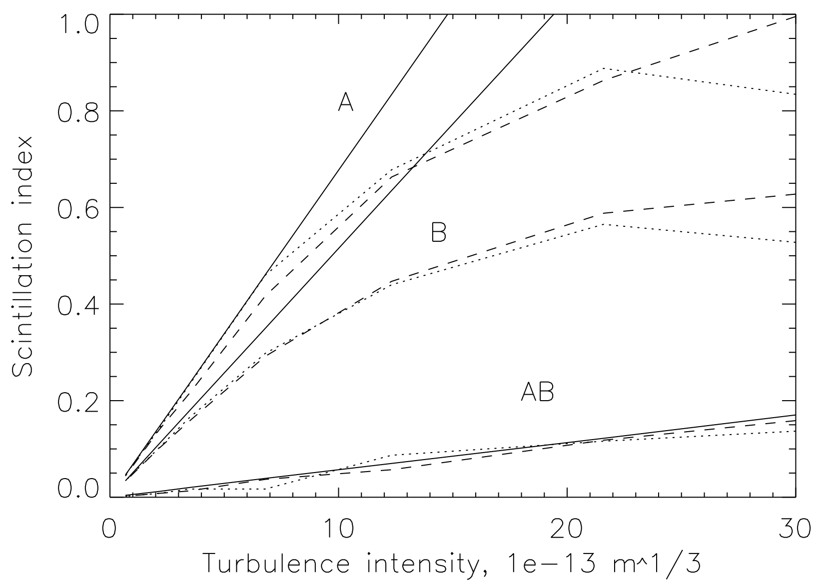

Figure 1: Scintillation indices in the small 2-cm aperture (A), ring aperture (B) and differential (AB) versus the strength of 8-km turbulent layer. Full line – theoretical, dashed line – linear formula, dotted line – logarithmic formula.

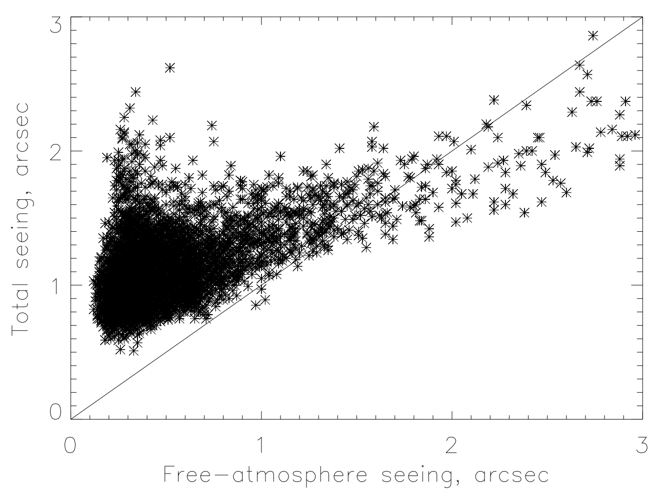

Figure 2: Relation between the "free-atmosphere seeing" measured by MASS and the total seeing measured by DIMM in January 2003 at Cerro Pachón in Chile.

phenomenon is observed (Fig. 2). When bad seeing is caused by high layers, MASS typically overestimates it in comparison to DIMM. A possible explanation may consist in the non-Kolmogorov turbulence spectrum under these conditions, with an excess of small-scale perturbations.