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# Measuring turbulence profile from scintillations of single stars

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**Abstract.** The new methods to extract the information on vertical turbulence profile from the spatial structure of single-star scintillations are reviewed. The first method involves quasi-simultaneous measurements of scintillation indices with several apertures of different size. It permits to derive isoplanatic angle and to obtain approximate estimates of free atmosphere seeing and effective turbulence altitude by adjusting a single-layer model to the data. The second method consists in measuring the differential scintillation index in a pair of small concentric apertures. It gives the integrated free-atmosphere turbulence directly. Both methods were used at the Maidanak Observatory in 1998-99. The median contribution of free atmosphere to the total turbulence integral at Maidanak is estimated to be 0.38.

Based on this experience, we plan to build the Multi-Aperture Scintillation Sensor (MASS) - a turbulence profiler with 4 concentric apertures. About 6 independent points on turbulence profile will be measured. Good estimates of seeing, isoplanatic angle, and, possibly, atmospheric time constant will be obtained as well. MASS is conceived as a permanent automatic profile monitor to provide real-time data in support of adaptive optics and interferometry operations.

## 1. Introduction

The knowledge of the turbulence vertical profile $C_n^2(h)$ is required to compute the performance of adaptive optics (AO) systems (both with natural and with laser guide stars) and to estimate the differential fringe motion and astrometric precision of stellar interferometers. This information is also vital for understanding the physics of seeing and for selecting sites for future Extremely Large Telescopes, which are believed to rely heavily on AO.

Existing techniques for $C_n^2(h)$ measurements include a direct *in situ* balloon micro-thermal sounding and a remote optical sounding with double stars, known as SCIDAR (Avila et al. 1997). Both methods are not adapted for continuous profile monitoring. It is then not surprising that no solid statistical database of $C_n^2(h)$ exists for any site. Recognizing this need, we and some other researchers turn to the study of single-star turbulence profilers that would be suitable for monitoring.

From the theory of light propagation in turbulence (Roddier 1981 and references therein) it is known that the intensity and spatial structure of the scintillation pattern ("flying shadows") depends on the altitude of turbulent layers and their strength. Inverting this relation, it is possible to restore the $C_n^2(h)$ from the analysis of the scintillation spatial structure. This principle is basic to all single-star profilers, and only the type of data and its analysis change from one device to another.

Let $I(\vec{x})$ be the instantaneous light intensity in the telescope pupil plane, convolved with some instrument response function (e.g. pixel profile). The covariance of intensity fluctuations normalized by the square of the mean intensity, $C(\vec{r}) = \langle \Delta I(\vec{x}+\vec{r}) \Delta I(\vec{x}) \rangle / \langle I \rangle^2$, contains all the information on scintillation spatial structure. Often only the normalized dispersion, or *scintillation index* $s = C(0)$ is measured instead of the full $C(\vec{r})$, but the theory remains the same.

The relation between rotationally-symmetric $C(r)$ and $C_n^2(h)$ which follows from the wave propagation theory for the case of small fluctuations is given by

$$C(r) = \int_0^\infty \mathrm{d}h C_n^2(h) W(h,r), \tag{1}$$

$$W(h,r) = 0.033 \cdot 16\pi^2 k^2 \int_0^\infty \mathrm{d}f f^{-8/3} \sin^2(f^2 h/2k) J_0(fr) A(f), \tag{2}$$

where $k = 2\pi/\lambda$ is the wavenumber, $f$ is the modulus of spatial frequency, $J_0$ is the Bessel function, and $A(f)$ is the squared normalized Fourier transform of the aperture (detector pixel) function, averaged in angle if not circular-symmetric.

Thus, any scintillation measurement leads to an integral of $C_n^2(h)$ multiplied by some *weighting function* (WF) $W(h)$. It is this relation that needs to be inverted to get $C_n^2(h)$, and the more peaked $W(h)$ is, the better is the altitude resolution of the profiler. Unfortunately, in all cases $W(h)$ is rather smooth, which means that all single-star profilers have intrinsically low resolution.

The characteristic size of the scintillation pattern is few centimeters, being governed by the Fresnel radius $\sqrt{\lambda h}$. Hence, only the high-frequency part of the turbulence spectrum contributes to the scintillation. On the other hand, an image degradation in large telescopes is caused mainly by the $r_0$-sized eddies, and the Fried parameter $r_0$ reaches 1 m in the infrared. This means that a theoretical form of the turbulence spectrum (Kolmogorov law) must be used to extrapolate the scintillation-based measurements to large telescopes. This problem is common to other standard techniques like SCIDAR and, to smaller extent, DIMM. Comparing further the scintillation profilers with DIMM, we note that the latter can have longer exposure times because it is sensitive to eddies of larger spatial scales. Hence, the problem of temporal bias in scintillation profilers must be addressed carefully.

## 2. Overview of the techniques

The first application of the above principle was that of Ochs et al. (1976), who have built the "scintillometer". The Fourier transform of the scintillation pattern was measured, with an idea to use the oscillating nature of the $\sin^2(f^2h/2k)$

term in (2) and to obtain the WFs with the best possible resolution. However, the oscillations are wavelength-dependent, hence smoothed when a wide spectral passband is used. The resolution achieved was of the order of $\Delta h/h \approx 1$. Moreover, as shown by Krause et al. (1993), instrument calibration strongly depends on the spectral type of a stellar source.

The idea to measure the correlation function $C(r)$ and to extract the profile from it was formulated repeatedly in the literature (e.g. Peskoff, 1968). In essence it is equivalent to the Fourier approach mentioned above. With modern CCD detectors and suitable inversion techniques, it is expected that altitude resolution of $\Delta h/h \approx 0.5$ can be achieved, although it will be necessary to measure $C(r)$ with a relative accuracy of $10^{-4}$ (Tokovinin 1998a). A careful study of detector and its noise characteristics will be essential for this method, which still awaits implementation.

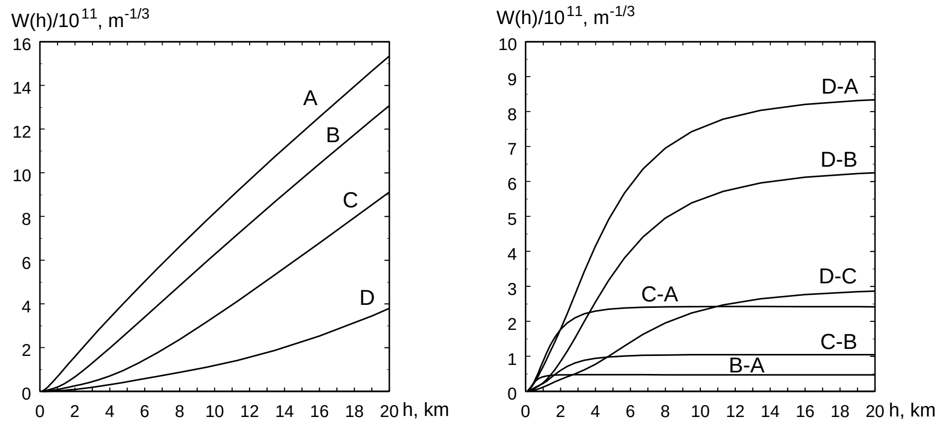


Figure 1. The weighting functions $W(h)$. Left (a): WFs corresponding to the normal scintillation indices for a set of 4 concentric apertures A, B, C, D with the outer diameters of 2.0, 3.7, 7.0, and 13.0 cm. Aperture A is circular, other apertures are annular. Right (b): WFs for pairwise differential combinations of the same apertures.

Measurement of scintillation indices with circular apertures of different diameters can be viewed as a substitute for the full $C(r)$ measurement. The practical advantage is that only a small number of light detectors is needed. If photo-multipliers are used, the statistical properties of photon counts can be studied with a high precision and taken into account in the data reduction – an essential requirement for unbiased measurements. It is important to take the data with different apertures simultaneously, otherwise the temporal variation of turbulence will hide the subtle differences between the indices. This method was tried by us (see below). Weighting functions for a set of 4 circular apertures are plotted in Fig. 1a.

The new concept of *differential* scintillation index $s_d = \langle (\Delta \ln I_1/I_2)^2 \rangle$ for a pair of apertures was introduced in (Tokovinin 1998b). Here $I_1$ and $I_2$ are the intensities of light detected in the two apertures *simultaneously*. Like the normal indices, the differential ones are calculated from (1) and (2), only the aperture function $A(f)$ needs to be modified. For a combination of concentric circular and annular apertures the $W(h)$ saturates at altitudes $h > d^2/\lambda$, i.e. when the diameter of the outer aperture $d$ becomes less then the Fresnel radius $\sqrt{\lambda h}$ (Fig. 1b). This very desirable property of differential indices can be exploited for profile measurements, as demonstrated below.

All scintillation-based techniques are intrinsically insensitive to turbulence at the ground layer: the WFs go to zero there. However, this can be overcome if the detector is optically conjugated to some negative altitude $-H$, adding a virtual turbulence-free propagation path to the real atmosphere. All WFs are then shifted to the left, and ground turbulence can be sensed. Pioneered in the Generalized SCIDAR (Avila et al. 1997), this idea can be used also with single stars, and can be combined with the approaches listed above.

If a small aperture is conjugated to a positive altitude $H$, the WF is approximately proportional to $|h - H|$: a turbulent layer at $h - H$ does not contribute to the scintillation, while other layers do. Changing $H$ rapidly, we may scan the whole range and use the "sharp" angle of $W(h)$ to restore the profile. This approach promises a potentially better resolution, and was evaluated in (Tokovinin 1998a). However, for large $H$ the strong ground-layer turbulence would produce strong scintillation ($s > 0.3$), and the formulas would no longer be valid. If the ground layer alone corresponds to a $0.5''$ seeing, the scintillation saturates at $H \approx 15$ km. Large conjugation heights are uncomfortable also for practical reasons, hence this interesting approach seems to be of limited use. It might be valuable, however, for detailed measurements of turbulence profile at low altitudes.

Another important atmospheric parameter, the time constant $\tau$, is related the $C_n^2(h)$-weighted integral of wind velocity modulus $|\vec{V}(h)|^{5/3}$ (Roddier, 1981). If the wind is known, it can be computed from the $C_n^2(h)$. However, the temporal spectrum of scintillations contains some information on wind, and we are developing a method to estimate $\tau$ directly from the light flux variations. Thus a profiler can potentially deliver the complete set of atmospheric parameters: seeing, isoplanatic angle, and time constant.

## 3. Practical experience at Maidanak

Scintillation measurements were carried out at the Maidanak Observatory in Uzbekistan in 1998-99 in a framework of an international project. The final report (Sarazin, 2000) contains complete results of this project, some of which are also published by Ehgamberdiev et al. (2000).

The first set of data was obtained with a 4-channel photometer, and consists of quasi-simultaneous measurements of scintillation indices with 10 (or less) apertures with projected diameters from 1.35 to 15.1 cm. Light from a bright star was detected simultaneously in 3 colors (blue, yellow and red) with an exposure time of 1 ms.

Combining the scintillation indices, it is possible to estimate the characteristic height $H_{\mathrm{eff}}$ of the dominating turbulent layers in the free atmosphere and their total energy $E$ (hence, seeing). For example, the index in a large aperture is approximately proportional to $H_{\mathrm{eff}}^2$, while in a small aperture it is proportional to $H_{\mathrm{eff}}$; the ratio of these indices leads to $H_{\mathrm{eff}}$, and the total turbulent energy $E$

is then obtained from $W(H_{\rm eff})$. Alternatively, a good measure of the turbulence integral in the free atmosphere is obtained from a linear combination of the indices in the two smallest apertures. Both techniques lead to similar results. The whole free atmosphere is thus represented by a single "equivalent layer". This approach and the results are further described in Kornilov & Tokovinin (2000).

A prototype instrument with two concentric apertures (Double-Aperture Scintillation Sensor, DASS) was also operated at Maidanak in 1998 during 4 nights. The "flat" weighting functions of differential scintillation provided good estimates of the integrated free-atmosphere turbulence $E$; equivalent altitude was estimated by comparing the differential and non-differential indices.

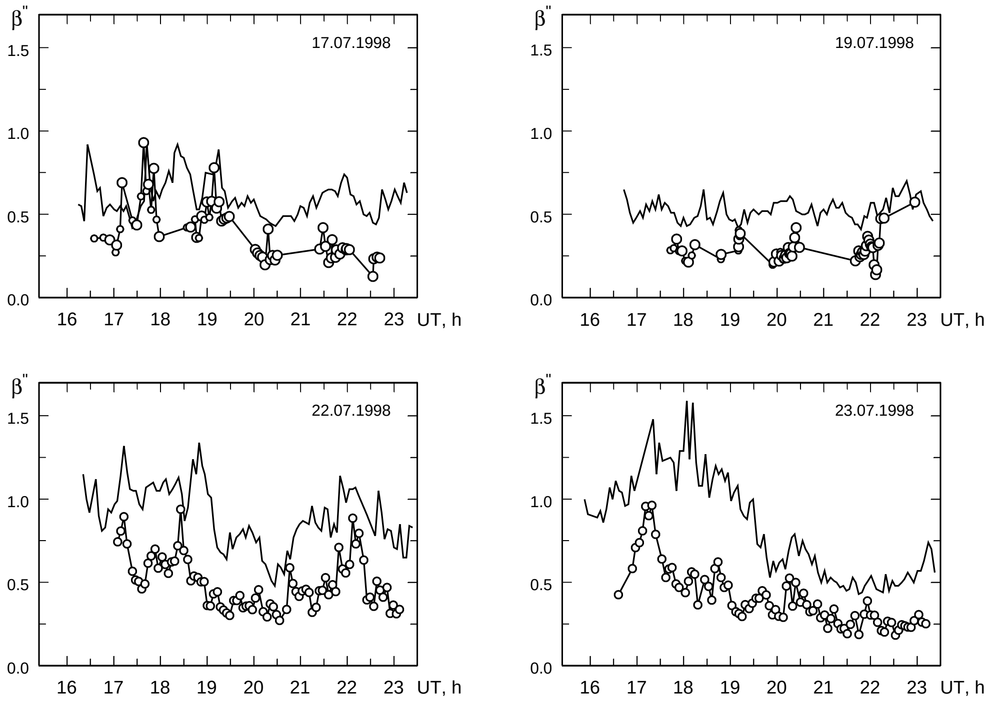


Figure 2. Simultaneous measurements of seeing in July 1998: total seeing as a finction of time (full line) and the free atmosphere seeing $\beta_{\rm free}$ (circles). Data on $\beta_{\rm free}$ come from DASS (two upper plots) and from the 4-channel photometer (two lower plots).

In Fig. 2 the seeing in the free atmosphere $\beta_{\rm free}$, as measured from scintillations, is compared to the total seeing $\beta$ simultaneously measured with the GSM instrument (Martin et al. 1998). It is evident that $\beta_{\rm free}$ is almost always smaller than $\beta$, although still non-negligible. However, during the short periods of time when the free atmosphere dominates, $\beta_{\rm free} \approx \beta$. The remarkable occasional agreement between the instruments (GSM and profilers) which use completely different principles is a strong (although indirect) evidence that our approach is valid.

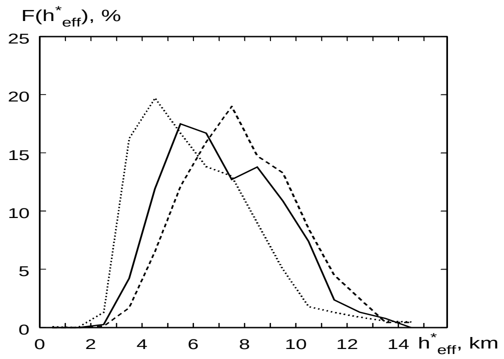


Figure 3. The distribution of the effective altitude of turbulence in the free atmosphere above Maidanak for 3 seasons: summer 1998 (full line), summer 1999 (dashed line), autumn 1999 (dotted line).

Simultaneous measurements of total and free-atmosphere seeing were conducted during 3 observing sets (a total of 34 nights). The distribution of $H_{\rm eff}$ for those sets is shown in Fig. 3. A bimodal character is apparent, with dominating altitudes either at tropopause (9 km) or around 3-5 km above the mountain. Appearance of a strong 3-km layer was frequently associated with overall seeing degradation; this layer must be caused by some large-scale orographic features of the Maidanak region.

From the comparison of the free atmosphere and total seeing it follows that the median contribution of the free atmosphere above Maidanak to the total integral is 38%, the remaining 62% being generated in the boundary layer. If the boundary layer only were compensated by adaptive optics, it is expected that the median seeing would improve from $0.70''$ to $0.39''$.

## 4. Multi-aperture profiler – MASS

The differential and normal scintillation indices for a set of 4 circular/annular concentric apertures permit to sample the $C_n^2(h)$ profile with very different WFs (Fig. 1). This information is about all that can be derived from single-star scintillations, so a 4-aperture instrument (called Multi-Aperture Scintillation Sensor, or MASS) offers an interesting practical solution for a single-star profiler.

A difference of two "step-like" WFs produces a new WF with a peak at some altitude. Generalizing, it is possible to obtain a set of WFs peaking at different altitudes as linear combinations of the initial WFs, and hence to measure the profile with low resolution. If an estimate of some integral turbulence characteristic (e.g. isoplanatic angle, free-atmosphere seeing) is needed, the same approach is used to obtain an approximation to the desired WF (e.g. proportional to $h^{5/3}$ or step-like) as a linear combination of the instrumental WFs.

Representing a profile by a combination of 6 layers at fixed altitudes, it is possible to select the layer intensities (subject to a positivity constraint) that agree best with the measured indices. Fig. 4 shows the results of such a restoration of the simulated data. The relative noise of indices was 1%, corresponding roughly to 1 minute integration on bright stars. The precision achieved is already better than the natural profile variability because of intermittency. The integral of $C_n^2(h)$ is measured with a precision of about 1%.

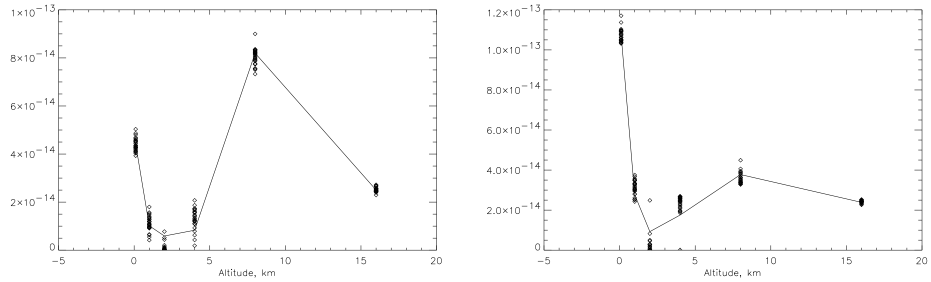


Figure 4. Numerical modeling of profile restoration from the MASS data. Two real profiles at Paranal were re-binned into the 6 layers, their energies in $m^{1/3}$ are shown by a full line. Circles show the results of profile restorations with 30 noise realizations from a set of 8 indices: 4 normal, 3 differential, and one differential in generalized mode.

MASS is insensitive to low turbulence, unless a generalized regime is used. Pupil conjugation to $H = -1$ km is sufficient to recover the boundary layer and to measure the total profile. The WF shifted by $-1$ km for the pair of the smallest apertures B and A is almost flat, and corresponding differential scintillation index can be directly converted to the total seeing.

Practically, measurements in the generalized and normal regimes will be interlaced, with a switching time of 1 s and a total accumulation time of 1 min. or more. The extended set of indices thus obtained is quasi-simultaneous, and improves considerably the profile reconstruction capabilities at low altitudes. However, the generalized mode is more difficult to operate experimentally for two reasons. First, the pointing errors, which are irrelevant in the classical regime, become troublesome, as they provoke a motion of the conjugated apertures over telescope pupil. The pupil diameter must exceed the aperture diameter by at least $2\sqrt{\lambda |H|} + 2\alpha |H|$, where the first term accounts for the Fresnel diffraction at the pupil edge, and the second term arises from the tracking errors of $\pm\alpha$. With a telescope size of 15 cm, $H = -1$ km, and B aperture diameter of 3.7 cm, this leaves a tracking tolerance of $\pm 7''$, which might pose some problems in portable instruments.

A second practical problem is the sensitivity of the generalized regime to telescope aberrations. For example, even a small variation of telescope focus changes the conjugation altitude considerably, hence the focus must be controlled with a high accuracy. Aberrations other than defocus can not be tolerated either, otherwise the conditions of the generalized mode are violated. In short, a bad and cheap telescope is sufficient for the normal MASS operation, wheras a telescope with good tracking and diffraction-limited optics at a pupil scale of few cm (image quality of $2''$ or better) is needed for the generalized mode. Still, these requirements do not extend beyond typical specifications for normal imaging telescopes, and we are certain that a *portable* version of MASS working in the generalized regime can be built.

When the MASS is built and tested, we may start thinking of a robotic instrument for seeing and profile measurements. One of the possible concepts is shown in Fig. 5. Telescope with a mounting is replaced here by a flat celostat

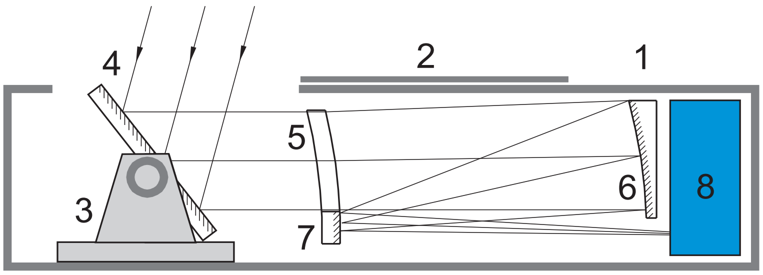


Figure 5. A concept of the robotic profiler. The box 1 with a sliding cover 2 houses all elements and protects the instrument from wind, rain, and dust. The light from a near-zenith star falls onto the flat mirror 4 on alt-azimuth mount 3, goes through the meniscus 5, is reflected by a spherical mirror 6 and a secondary 7 and enters into the detector module 8.

mirror and a stationary Maksutov-type off-axis optics. The celostat is kept fixed for the periods of 10-30 seconds, while data are taken. During this time stellar drift remains small enough even in the generalized mode. Then the pointing is updated and measurements are resumed. The outer box protects the instrument and facilitates its transport and installation. The instrument must be designed to operate unassisted for long periods of time. If this dream is realized, turbulence measurements might become as simple and routine as air temperature monitoring!

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