The Distribution of Mass Ratio in Late-type Main-sequence Binary Systems

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Abstract

The maximum likelihood method used to derive the distribution of mass ratio q from the combined data on single and double-lined binaries is described. It's validity is demonstrated by numerical simulations. The results of the radial velocity survey of nearby K and M dwarfs are presented, the extended KM-dwarf sample and the sample of nearby G-dwarfs are analyzed as well. The distributions of q in these 3 samples are flat and the differences between them appear to be not significant. However the frequency of spectroscopic binaries with periods less than 3000 days among K and M dwarfs is lower than for G dwarfs and the cutoff in the secondary mass distribution at 0.08 solar mass is seen.

1. Introduction.

The radial velocity survey of a well chosen sample of stars can provide answers to the following questions:

– What is the frequency of short-period binaries?

– What is the shape of the mass ratio distribution?

– Are there any substellar mass secondary companions?

In the analysis of the data provided by such a survey it is necessary to consider various selection effects and the influence of random orbital inclinations. The data analysis scheme based on the maximum likelihood principle will be explained first. Then three samples of nearby dwarf stars will de described and the distributions of the mass ratio q in these samples will be discussed.

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2. Derivation of mass ratio distribution.

The best estimate of the distribution of the mass ratio $P_q(q)$ is found by maximizing the probability L of given observations. It is called the likelihood function and it is equal to the product of the probabilities P_i of the observations of i-th star. All information contained in the observations is eventually expressed by these quantities P_i.

Radial velocity observations of a homogeneous sample of stars carried out over a sufficiently long time lead to the discovery of a number of binary systems and, eventually, to the determinations of their orbits. It is natural to place un upper limit to the orbital periods taken into consideration in order to assure reasonable detection limits. For the single-lined binaries (SB1) only mass functions are known. If the mass of the primary is estimated from its color the mass function can be replaced by the equivalent quantity $F = qx(1+q)^{-2/3}$ where $x = \sin i$. For SB2 the mass ratio q is measured directly.

The combined analysis of SB1 and SB2 data requires some caution. The probability of non-detection of the lines of secondary components depends on the mass ratio q and is modelled by the function $d(q)$ which goes from 1 at $q = 0$ to 0 at $q = 1$. The probabilities D_{SB1} and D_{SB2} for a spectroscopic binary to be single- or double-lined are expressed as integrals over q of mass ratio distribution P_q multiplied by d or $1-d$.

We have chosen to represent the transition from non-detection to complete detection by the linear drop within the q-bin that contains the lowest-q SB2 in a given sample. The numeric simulations show that incorrect modelling of SB2/SB1 partition produces in the restored distributions maxima and minima at the SB1/SB2 margin. Since the real process of secondary line detection can hardly be described by any simplistic model the features of q distributions around $q = 0.7$ should be regarded with suspicion. It is of course possible to treat all binaries as SB1 but this would inevitably entail some loss of information.

The distribution of mass ratios in the SB1 sub-sample is equal to $P_q(q)d(q)$. The distribution of the mass-function equivalent F is obtained by its convolution with the distribution of $x = \sin i$ that results from the hypothesis of random orientation of orbital planes:

$$P_F(F) = \frac{1}{F}\int_{q_{\min}}^{1} \frac{x^2}{\sqrt{1-x^2}} P_q(q)d(q)\mathrm{d}q \qquad (1$$

At any given F there is a relation between x and q. The lower limit of integration $q_{\min}$ corresponds to $x = 1$. The singularity is integrable and for practical purposes it is more convenient to carry the integration over x rather than over q.

The observation of i-th SB1 consists of two distinct facts: i) double lines are not seen and ii) the mass function is equal to F_i. The probability of observation P_i is equal to $P_F(F_i) \cdot D_{SB1}$. Similarly for SB2 it is equal to $(1 - d(q_i)) \cdot P_q(q_i) \cdot D_{SB2}$. Here the first multiplier can be safely omitted since it does not depend on the q-distribution. The normalization of P_i is of no importance. For the same reason the use of mass function or any other related quantity instead of F for SB1 would lead to the same result.

In practice the distribution is discretized into K bins, $P_q(q) = f_k$ for $q_{k-1} < q < q_k$. The probabilities P_i are then represented as scalar products, $P_i = \sum_k p_{ik} f_k$, and the natural logarithm of the likelihood function is

$$\ln L = \sum_{i=1}^{2N} \ln \sum_{k=1}^{K} p_{ik} f_k \qquad (2)$$

where N is the total number of spectroscopic binaries in the sample. The elements of the first N rows of the matrix $\{p_{ik}\}$ are easily calculated using (1) for SB1 and are equal to 0 or 1 for SB2. The elements of the next N rows come from the detection probabilities D_{SB1} or D_{SB2} and are equal to the integrals of d or $1 - d$ over k-th bin.

Each term in (2) should be divided by the probability of the detection of binarity to take into account undetected systems. This was not done here since detection probabilities for SB1 are always close to 1. The detection of SB1 was studied in (Tokovinin 1992 – hereafter [T92]) by numerical modelling. Virtually all SB2s with periods less than 3000 days are detected as well.

There exists a very efficient iterative technique for finding the maximum of (2) over $\{f_k\}$. It was described by Lucy [L74]. He also applied it to the correction of the observed distribution for the effects of random inclinations and noted its usefulness in a more general context of the solution of inverse problems. The iterative method of Mazeh and Goldberg [MaG92] for finding the distribution of q is similar to that of Lucy.

Inverse problems are well-known to be very sensitive to the noise in the input data. Our numeric simulations indeed show that the maximum likelihood solution is quite satisfactory for big samples (more than 50 stars per bin) but tend to have irregular maxima and minima when the sample is small. A similar behavior was noted in tomographic image restoration (Llacer and Nuñez [LlN90]).

In the real life the size of sample is usually small and some regularization prescription is necessary to produce a "good-looking" distribution. Lucy suggested to stop iterations well before the convergence. When the uniform distribution is taken at the first step this prescription gives a regularized solution which is biased towards uniform distribution.

A somewhat better approach seems to be the search of the most smooth distribution that is compatible with the observations. The natural degree of compatibility is offered by the likelihood function itself. If $\{f\}$ is the exact solution then any distribution $\{g\}$ is plausible if $\ln L(f) - \ln L(g) < 1$ since it means that $\{g\}$ would make the probability of our observations only e times smaller. The choice of 1 as a plausibility threshold is, of course, arbitrary.

We use the sum of $(f_k - f_{k-1})^2$ as the smoothness criterion and add it with some negative coefficient to $\ln L$. This coefficient is chosen by trial and error to give a solution with $\ln L$ that is smaller by 1 than its maximum value. Thus a distribution that is biased towards maximum smoothness but still compatible with the observations is obtained.

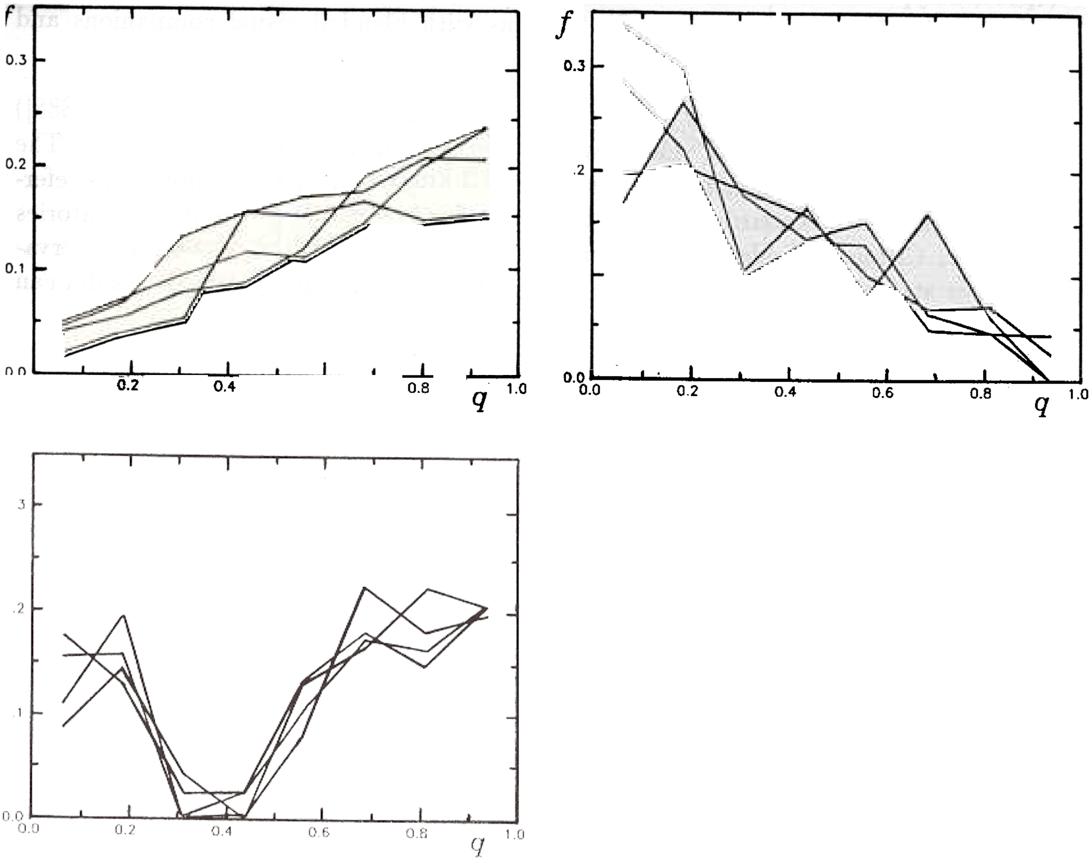

Figure 1. Results of the numeric simulations. Samples of 50 artificial binary stars with known distribution of q were generated and analysed: (a) linear growing distribution, (b) linear falling distribution, (c) uniform distribution with a gap at $q = 0.25 - 0.5$. Results for 4 realizations are plotted.

The results of numerical testing of the data analysis scheme described above are given in Fig. 1. The samples of 50 artificial binary stars with known q distributions were generated and then analyzed just as real data. Not only are the linearly rising and falling distributions well restored but the distribution with a sharp gap is also reasonably well recovered despite the fact that it is not at all smooth.

3. Radial velocity surveys of dwarf stars.

Our radial velocity survey of K and M dwarfs in the solar neighbourhood was started in 1985. The sample included stars from the Gliese catalogue [Gli69] and its supplement [GlJ79] with visual magnitude brighter than 10^m, $B - V$ color from 0.76 to 1.5 (spectral types from G9 to M3). Only limited part of the sky with $\delta > -10°$ and α in $14^h..0^h..5^h$ zone was surveyed. Systems with blended visual companions and subdwarfs were excluded. The total number of stars is 200.

The correlation radial-velocity meter (Tokovinin [Tok87]; Goryatchev et al. [GKS88]) was used to measure radial velocities with the typical precision of 0.5 km/s. The limiting accuracy of this instrument is about 0.3 km/s. Velocity zero point was referenced to the IAU standards. Several telescopes located at the different observatories (Moscow, Crimea, Maidanak, Abastumani) were used. On the average 7 observations per star have been made. More information on this survey and its results can be found in [T92]. The first 4 orbits are published in [Tok91].

The KM-dwarf sample contains 10 single-lined and 12 double-lined binaries with periods less than 3000 days. However 5 of SB2's are just at the magnitude limit of the sample and would not be included were they single stars. They were removed from further analysis.

The extended KM-dwarf sample is obtained by adding to all our data the results of recent radial velocity surveys: those of Hyades by Griffin et al. [GGZ85], a subdwarf survey of Latham et al. [LMC88] and some orbits published by Duquennoy and Mayor [DuM88a], [DuM88b]. Only stars with $B - V > 0.7$ and periods greater than 3000 days were selected.

The sample of G-dwarfs observed by Duquennoy and Mayor [DM91] with periods less than 3000 days was also analyzed. The general characteristics of these 3 samples are summarized in Table 1.

4. Results and discussion.

The restored distributions of mass ratio are shown on Fig. 2. The features around $q = 0.7$ are most likely not real but caused by the SB1/SB2 dichotomy. In all 3 samples the distribution of mass ratio looks surprisingly flat. The deficiency of binaries with the smallest q reflects the absence of substellar mass companions. A

Table 1.

Sample	Number of SB1	Number of SB2	Lowest q
Main KM	10	7	0.65
Extended KM	32	25	0.65
G-dwarfs	19	13	0.55

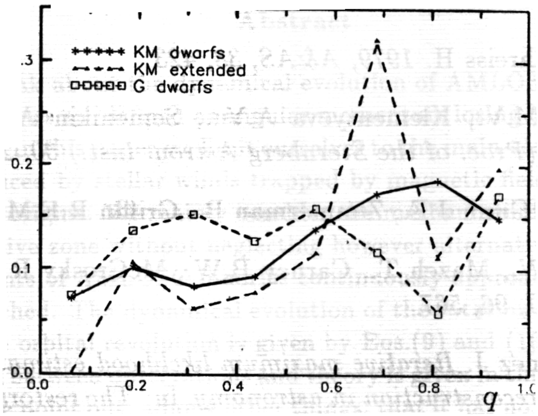

Figure 2. Distribution of mass ratio q in three samples of nearby dwarf binary stars with periods less than 3000 days.

more thorough analysis of their detection limits and the upper limits of their frequency are given in [T92]. The rise towards $q = 1$ in the extended KM sample can be explained by its heterogeneous nature and hence the influence of selection effects in favor of SB2.

It is now clear that the gap in the mass distribution of the secondary components at 0.2-0.3 solar mass that was "discovered" in [T92] is due to the incorrect treatment of SB1/SB2 selection effects together with the absence of any regularization.

The data on single stars must be included in the analysis if the estimation of binary frequency is needed. This was done in [T92] and will not be repeated here. It appears that the frequency of spectroscopic components with periods less than 3000 days for KM dwarfs ($10 \pm 2\%$) is less than for G dwarfs studied by [DM91] that is estimated to be 17% from their Fig.7. This fact may be related to the existence of lower limit of companion mass.

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