Atmospheric time constant measured from DIMM and MASS

M. Sarazin[1,a], A. Tokovinin[2], G. Lombardi[3], J. Navarrete[3]

[1]ESO, Garching, Germany
[2]CTIO, La Serena, Chile
[3]ESO, LaSilla-Paranal Observatory, Chile

Abstract. Here we adjust traditional definitions of atmospheric time constant and isoplanatic angle to make them more useful for adaptive optics and interferometry and, at the same time, easier to measure.

1. Introduction

Here we adjust traditional definitions of atmospheric time constant and isoplanatic angle to make them more useful for adaptive optics (AO) and interferometry and, at the same time, easier to measure.

1.1. The standard definition

The theory is based on the standard Kolmogorov turbulence model prescribing the "5/3" phase structure function $D_\varphi(r)$ as

$$D_\varphi(r) = 6.88 \left(\frac{r}{r_0}\right)^{5/3} \tag{1}$$

r_0 being the Fried parameter, either for the total propagation path or for a thin turbulent layer of integrated intensity $J = C_n^2 \Delta z$, where $r_0^{-5/3} = 0.423\,(2\pi/\lambda)^2 J$. The phase difference at two points separated by r is proportional to $r^{5/6}$. This leads to the standard definition of the atmospheric time constant τ_0, considering the frozen-flow hypothesis where the shift over time interval τ is proportional to the wind speed V, $r = V\tau$. Similarly, the anisoplanatic angle θ_0 is defined by noting that $r = \theta z$ for two sources at angular distance θ and a turbulence layer at a distance z from the telescope. Below we discuss the time-constant definition, but the argument fully applies to θ_0.

If the phase $\varphi(x)$ were a smooth function of coordinate x with a finite derivative a, we could write $\varphi(x + r) \sim \varphi(x) + ar$, in which case $D_\varphi(r) \sim <a^2> r^2$ at small r. For the Kolmogorov turbulence, however, the phase slope $<a>^2$ is infinite and the structure function is not quadratic (we neglect here the inner scale). The phase difference over a time τ caused by the whole atmosphere equals

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[a] e-mail : msarazin@eso.org

$$\sigma_\varphi^2(\tau) = \left(\frac{\tau}{\tau_0}\right)^{5/3} \quad . \tag{2}$$

The standard definition of the atmospheric time constant τ_0

$$\tau_0 = 0.314 \, \frac{r_0}{\bar{V}_{5/3}} \tag{3}$$

uses the wind-speed moment $\bar{V}_\mu$ with $\mu = 5/3$,

$$\bar{V}_\mu = \left[\frac{\int_0^\infty C_n^2(z)V^\mu(z)dz}{\int_0^\infty C_n^2(z)dz}\right]^{-\mu} \tag{4}$$

1.2. Adjusted definitions

There are two problems with the standard definition of τ_0. First, as noted by Stone et al. [1], for an aperture of diameter D the phase error tends to infinity at large τ or θ, while in reality the error in AO tends to a finite value: twice the total phase variance $1.03 \, (D/r_0)^{5/3}$ (the phase becomes totally un-correlated at large τ). This mismatch is traced to the fact that Eq. 2 includes the piston term which dominates at $r \gg D$ but is irrelevant for imaging and AO performance. The second problem occurs at small $r \ll D$, where the temporal error is caused by small-scale perturbations that are not corrected anyway. We focus here on this second, short-exposure regime.

The textbook formula for residual phase error in AO,

$$\sigma_{AO}^2 = \sigma_{noise}^2 + \sigma_{fit}^2 + \sigma_{time}^2 + \sigma_{aniso}^2 \tag{5}$$

(see e.g. Eq. 11.1 in the Chapter 11 of [2]), is misleading because it includes the effects of small-scale perturbations several times: in σ_{fit}^2 (where they should be) but also in the temporal and anisoplanatic terms.

In practice all quantities of interest (Zernike modes, Shack-Hartmann slopes, phase delay in an interferometer, etc.) are related to the spatially averaged phase. These quantities are smooth functions of coordinate with finite first derivatives. It means that corresponding structure functions are always quadratic for small arguments, $r \ll$ [averaging distance]. In this regime, the errors are expressed through $\mu = 2$ moments of the turbulence profile. Therefore, the second moment of the atmospheric wind speed $\bar{V}_2$ is more relevant than $\bar{V}_{5/3}$. Some examples follow.

- Zernike modes. Temporal spectra of Zernike modes were derived by Roddier et al. [3]. They note that "For finite n values and small τ values the curves are well approximated by quadratic laws. Hence for performance characterization one should use a quadratic law rather than a 5/3 law". The authors calculate quadratic errors for several modes. Kellerer & Tokovinin [4] derived quadratic formulae for defocus (mode 4) and piston (mode 1) in the short-exposure regime.

- Piston error in a long-baseline interferometer is well described by a quadratic formula for $\tau < 0.3D/V$, when errors from the individual apertures of diameter D are uncorrelated. To the first order, the piston temporal evolution is linear at short τ, being defined by the wave-front tilts at each aperture. The structure function of piston equals $(\tau/\tau_{piston})^2$, where the piston time constant $\tau_{piston} = 0.273(r_0/\bar{V}_2)(D/r_0)^{1/3}$[4]. The error of a fast fringe-tracking servo can be expressed through τ_{piston}. Note that on longer time scales $\tau > D/V$ the piston recovers its $\tau^{5/3}$ power law, but in this regime where turbulence outer scale and deviations from the frozen-flow hypothesis become important, this behavior is of academic interest only.

- Tilt makes no exception from the general quadratic trend. Tyler [5] gives quadratic formulae for temporal errors of Z- and G-tilts which depend on the 2nd wind moment $\bar{V}_2$. Analogous quadratic formula for tilt anisoplanatism can be found, e.g. in [2], Eqs. (12.60) and (12.61), Chapter 12. The tilt error depends on the 2nd moment of range $\bar{z}_2$ which is defined in analogy with $\bar{V}_2$. This same quantity enters the expression for atmospheric errors in differential astrometry [6].

- Scintillation in apertures larger than Fresnel radius depends on $\bar{z}_2$ (see [7], formula 8.12). The photometric error caused by scintillation depends on $\bar{z}_2$ at short exposures and on the C_n^2-weighted combination $\bar{z}_2/V$ at long exposures [8].

2. Measurements of $\bar{V}_2$

Given that essentially all turbulence-related quantities have quadratic temporal structure functions, the 2nd moment $\bar{V}_2$ can be measured by following their fast evolution – essentially by the speed of variation. The choice is a matter of convenience.

A working AO system records the wavefront evolution in time. The temporal structure function of any Zernike mode can serve to evaluate $\bar{V}_2$. Kellerer & Tokovinin [2] argue that defocus is the best choice, being the strongest and the slowest compared to other modes. This idea was tested on a 4-m telescope [10]. In contrast, the method of Fusco et al. [11] to estimate τ_0 at the VLT AO instrument NAOS is justified only for a single turbulent layer and does not deliver true wind-speed moments averaged over the whole atmosphere (Figure 1).
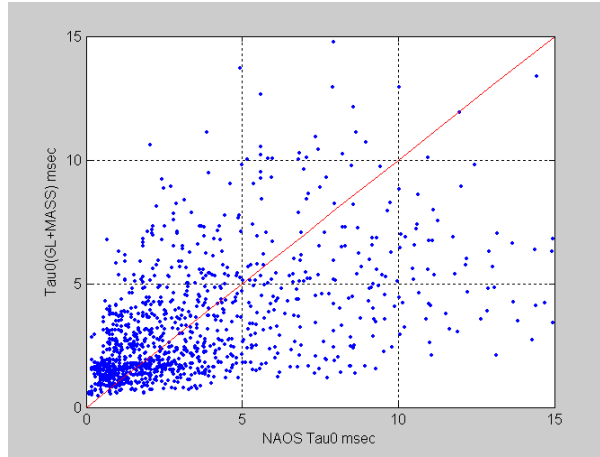
Fig. 1: Comparison of NAOS τ_0 with MASS–DIMM τ_0 for simultaneous measurements in 2004-2007. The following NAOS modes were used: 1-1 to 1-9 (except 1-7) & 2-1 to 2-3, total integration time >12s.

The amplitude of defocus variation is smaller and its evolution is faster at small apertures, making the measurement of $\bar{V}_2$ at small telescopes more challenging. Tokovinin et al. [9] tested a stand-alone time constant monitor, FADE, using a 36-cm telescope.

Scintillation of single bright stars was used for rough estimation of τ_0 in the MASS instrument [12]. This approximate method has been recently improved by Kornilov [13], who developed the short-exposure (quadratic) approximation for extracting consistent estimates of $\bar{V}_2$. This method was verified by comparison to wind vertical velocity profiles from meteorological models [14].

At present τ_0 at Paranal observatory is empirically derived from a combination of DIMM seeing and wind velocity at 200mb and at the ground [15]. To compute the exact value of τ_0 from MASS and DIMM C_n^2 profiles, wind velocity vertical profiles would be needed in real time, e.g. from an operational meteorological model [16]. A feasibility study is underway foe VLT and E-ELT Observatories. In the frame of the current upgrade of the VLT Astronomical Site Monitor, it is nevertheless foreseen to make use of MASS measurements for estimating the atmospheric time constant: this requires combining MASS estimates of $\bar{V}_2$ with some estimate of the contribution of the first few hundred meters above ground to which MASS is not sensitive. This ground layer has a total turbulence J_{GL} equal to the difference of DIMM and MASS turbulence integrals and, in a first approximation, moves at constant speed equal to the velocity of the wind measured at 30m on a meteorological mast nearby.

$$\bar{V}_2 = \left[\frac{J_{GL}(V_{30m})^2 + J_{MASS}(\bar{V}_{2MASS})^2}{J_{DIMM}} \right]^{1/2} \tag{6}$$

The statistics of $\bar{V}_2$ for Paranal are presented in Figure 2 with median values of 10m/s overall, 8m/s at the ground and 11m/s in the higher layers. The latter show, as expected, more frequent

occurrences of high velocities related to the presence of the jet stream above the observatory. The wavefront coherence time computed using $\bar{V}_2$ instead of $\bar{V}_{5/3}$ are presented in Figure 3, with median values of 2.4ms overall, 5.3ms in the ground layer and 3.6ms in the higher layers. Finally, the wavefront coherence time computed using $\bar{V}_2$ is compared in Figure 4 to the empirical algorithm using only DIMM [15] for the year 2010 at Paranal with median coherence time respectively 2.3ms and 1.9ms.
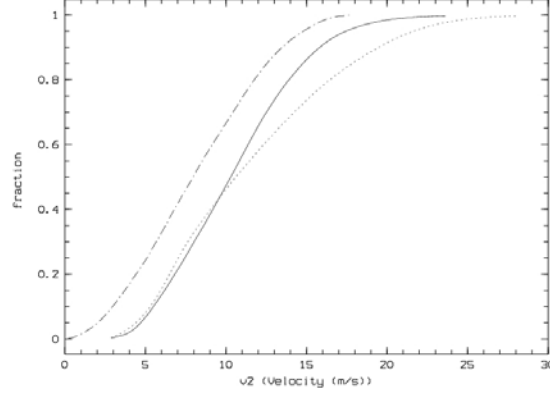

Fig. 2: Statistics of wavefront velocity second moments at Paranal (2005-2011): $\bar{V}_2$ (plain), $\bar{V}_{2MASS}$ (dots) and V_{30m} (dash)
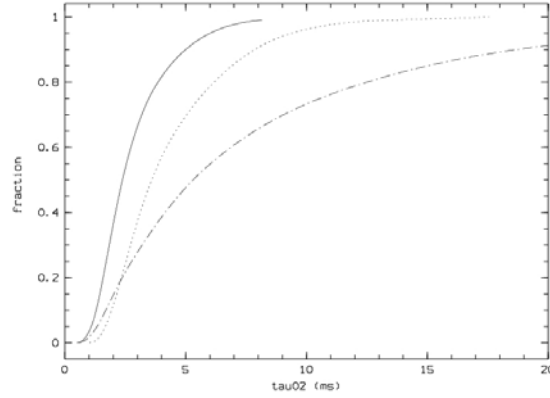

Fig. 3: Statistics of wavefront coherence time at Paranal (2005-2011) computed using the second moment velocities of Figure 2: full path (plain), high layers (dots) and ground layer (dash)
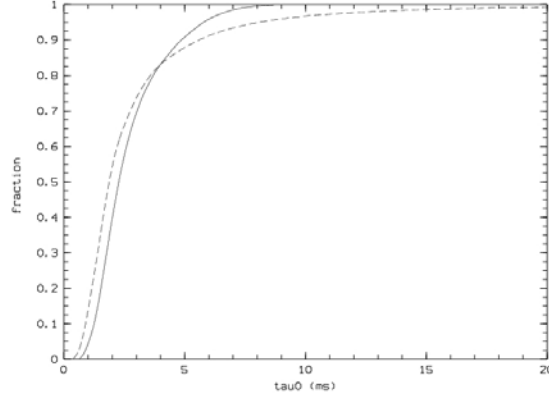
Fig. 4: Comparison of the statistics of the coherence time computed from $\bar{V}_2$ (full line) to tau0 empirically derived from DIMM [15] (dashes) in 2010 at Paranal

3. Conclusion

The standard Fried's definitions of τ_0 and θ_0 are unsurpassed by their elegance, involving combinations of 5/3 moments with r_0 in a system-independent way and appealing directly to the first principles of turbulence theory. However, the 2nd moments are preferable for both performance predictions and measurement (those two aspects are, of course, related). As a down-side, note that the formula for atmospheric effects with second moments become a bit more complicated and include some system characteristics, e.g. the aperture diameter D. On the other hand, unlike (5), the correct formulae with $\bar{V}_2$ or $\bar{z}_2$ are no longer misleading. The Fried's parameters τ_0 and θ_0 have dimensions of time and angle, respectively. However, they depend implicitly on the wavelength λ. We propose to replace them with the atmospheric moments $\bar{V}_2$ (in m/s) and $\bar{z}_2$ (in m). These quantities do not depend on λ. $\bar{V}_2$ is a truly system-independent integrated atmospheric parameter describing temporal evolution of turbulence-related quantities in the short-exposure regime.

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