Formation of Binary Stars
ASP Conference Series, Vol. 000, *2000*
R. D. Mathieu and H. Zinnecker, eds.

Statistics of multiple stars: some clues to formation mechanisms

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Abstract. The available information on the statistics of high multiplicity (3-6 components) systems is reviewed. The ratio of triple to binary systems is $f_3 \approx 0.11$, while $f_n \approx 0.25$ for higher n. Despite selection effects in the multiple star catalogue, the signatures of formation mechanisms are found in the distributions of period ratios and mass ratios. For example, the frequent occurrence of close sub-systems with periods less than 6 days can be explained by tidal dissipation in a 3-body system. In triple stars the angular momentum vectors of inner orbits are inclined to those of outer orbits by an average angle of 50°, hence the orbital spins are neither co-aligned nor completely random. Close binaries have a tendency to be found in higher-multiplicity systems, showing that close and wide binarity is statistically related. Future theoretical and observational studies are outlined.

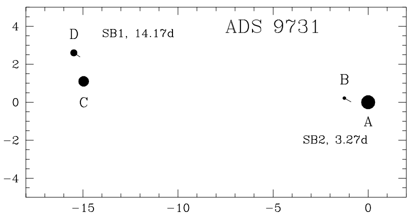
1. Introduction

Multiple stars can provide some insights into binary formation mechanisms, as pointed out by Batten (1973). The statistics of multiple stars is very scarce, despite some published works in this field. In an effort to systematize the available data, I compiled the first Multiple Star Catalogue (MSC) (Tokovinin 1997a). Of course, the statistics of the *catalogue* must not be mixed with a statistics of real objects, which should be obtained from well-defined distance-limited samples. Such works have been done for binary stars, but not yet for higher-multiplicity objects. Despite biases, the MSC does contain some information on multiple star statistics, and our approach is to find in a qualitative way which statistical features bear traces of formation and evolution of multiple stars.

2. How to measure stellar multiplicity?

To introduce the subject, the example of a sextuple system is given in Fig. 1. All multiples are hierarchical, naturally, because only hierarchical systems can be dynamically stable or at least meta-stable. So, multiples are represented by level (or mobile, or binary tree) diagrams.

The four visual components ABCD of ADS 9731 are listed in the WDS catalogue. The fifth close subsystem CE was announced, but not confirmed



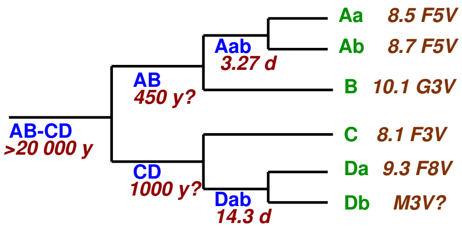


Figure 1. The example of a sextuple system ADS 9731. The position of the four visual components on the sky is mapped on the left (short lines from B and C denote their trajectories since discovery). On the right the hierarchical structure of this system is represented by a binary tree diagram.

later by repeated speckle observations. When radial velocities were measured (Tokovinin et al. 1998), it has been established that A and D are spectroscopic binaries, and hence ADS 9731 is sextuple.

Can we be sure that *all* components are now known? Looking at the level diagram, we see that there are still two available low levels in the hierarchy: both C and B can be close binaries. The first possibility seems to be ruled out by the combined results of speckle-interferometry and spectroscopy. However, B is too faint and too close to A, its radial velocity can only be measured with adaptive optics; hence, it can still be a close binary. Available photometry of B is not sufficient to eliminate this possibility. Additional faint components can be "placed" in this system at intermediate levels, e.g. in an orbit around Aab with a period of few tens of years. The criteria of dynamical stability do not forbid such companions, neither does photometry. Long-term radial velocity observations can, however, either reveal or rule out these subsystems.

This example is given to illustrate the fact that a combination of different observing techniques is needed to discover and study multiple stars. When this condition is met, dynamical constraints can be used additionally to prove that all components are indeed found, and the true multiplicity is hence known. This can be done only in exceptional cases. For a vast majority of the catalogued multiple stars, including ADS 9731, even the major parameter, multiplicity, is not yet known!

3. Overview of high-multiplicity statistics

3.1. Multiplicities

The actual (updated) version of MSC contains 807 physical systems. Their distribution over multiplicity is given in Table 1.

In Fig. 2 the total number N of catalogued systems of different multiplicity within a given distance d is plotted in log-log scale. The $N \propto d^3$ relation is expected, shown as full line. Actual growth is much slower, indicating that MSC is incomplete even within few parsecs. Recent discovery of new multiples within 10 pc (Delfossse et al. 1999) is a good illustration of this point. MSC incompleteness is due to a complex combination of discovery biases. The discovery

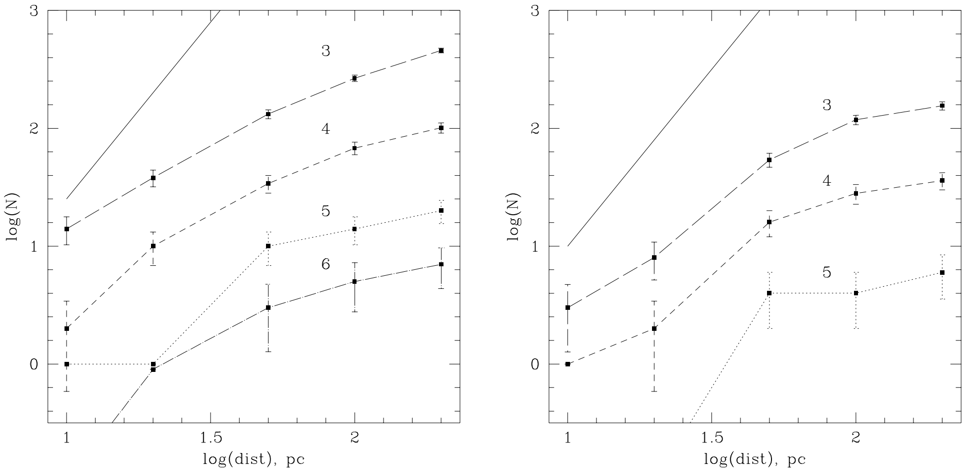


Figure 2. Number of systems N of different multiplicities catalogued in MSC within a given distance d (left). Right - same plot for solar-mass primaries only. Full lines show the $N \propto d^3$ slope.

is more complete for G-type stars. In Fig. 2 the same data are also plotted for systems with primary mass from 0.8 to 1.3 $M_\odot$, constituting about 30% of the whole sample. The slope of $N(d)$ is higher, but still less than 3.

The relative fraction of different multiplicities is roughly independent of sampling distance: the curves in Fig. 2 are parallel to each other. It is tempting to speculate that, despite severe discovery bias, this fraction reflects the *true* multiplicity ratio, because the discovery of different systems is affected in the same way. Multiplicity fraction $f_n = N_n : N_{n-1} \approx 0.25$ as estimated in the book of Batten (1973) is not very different from the values in Table 1, despite the greatly increased number of objects.

The fraction of triple stars relative to binaries, f_3, can be estimated as 0.11 ± 0.04 from the sample of Duquennoy & Mayor (1991) (DM91). It is significantly smaller than f_n for higher n. The same conclusion emerges from the analysis of some other samples. Moreover, the tendency of increasing f_n with n can be guessed. So, the number of high multiplicity systems decreases slower than n^{-f} if f were constant. Extrapolating this to higher n, one might predict that MSC should contain at least one seventuple. Actually two $n = 7$ systems are listed

Table 1. Multiplicity statistics in MSC

Triples	626	$(N_3 : N_2 = 0.11 \pm 0.03)$
Quadruples	141	$N_4 : N_3 = 0.22 \pm 0.02$
Quintuples	28	$N_5 : N_4 = 0.20 \pm 0.04$
Sextuples	10	$N_6 : N_5 = 0.36 \pm 0.14$
Total	807	

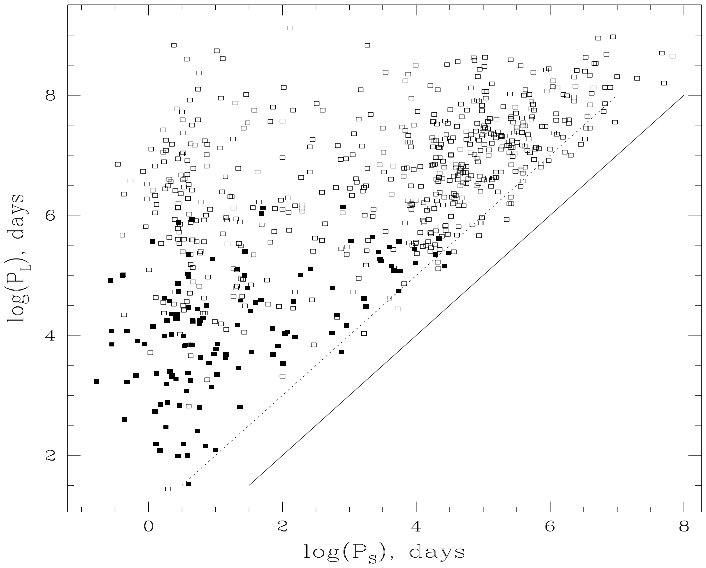


Figure 3. Periods at adjacent hierarchical levels, P_L and P_S, in days. Filled symbols correspond to both periods from orbital solutions, whereas for empty symbols at least one period is estimated from apparent component separation, parallax and total mass. Full line corresponds to equal periods, dotted line to $P_L/P_S = 10$.

(ν Sco and AR Cas), but both still need confirmation for some components. Undoubtedly, new observations will soon lead to discovery of $n = 7$ systems, but much larger samples are needed to search for $n > 7$.

It is interesting to study the relative frequency of the systems with same multiplicity but different hierarchy. This can be meaningfully done only for quadruples, by comparing the number of "binary + binary" systems with the number of "binary + wide companion + still wider companion" quadruples. These numbers in MSC turn out to be similar, 71 and 70. If a quadruple is formed by addig a new close binary to a triple, it seems that with equal probability the new binary can be added either to the tertiary, or to the inner binary.

3.2. Period statistics

The relation between the "long" and "short" periods at adjacent hierarchy levels, P_L and P_S, is given in Fig. 3. Future discovery of intermediate-level subsystems may significantly modify this plot, eliminating large P_L/P_S ratios. As it stands now, all possible period combinations are found. The smallest period ratio P_L/P_S is about 7, providing an empirical criterion of dynamical stability of 3-body systems which is in agreement with theoretical studies. Apart from this, there is no preferred period ratio.

The distribution of the short periods P_S is markedly non-uniform, showing a reduced number of objects with P_S between 6 days and 30 years. The existence of this partially filled gap was already noticed by Duquennoy & Mayor (1986). Of course, it can be nicely explained by discovery biases, because in this period range both radial velocity and direct resolution are difficult. The fall-off from the short period side at $P_S > 6^d$ is, however, too sharp. This was discussed in (Tokovinin 1999) and confirmed by an independent sample of newly discovered sub-systems. A physical mechanism that could be responsible for this feature is discussed in Sect. 4.1.

3.3. Coplanarity

Relative orientation of inner and outer orbits in a triple (or higher-multiplicity) system may be an important diagnostic of their formation. More precisely, the statistics of the angle ϕ between angular momenta of the orbits must be determined. Of course, ϕ changes in course of the dynamical evolution of an n-body system. However, to the first approximation this evolution is mostly a precession of an inner orbit around the total angular momentum vector, with almost constant ϕ. In some triple systems this precession has been actually detected (Mayor & Mazeh 1987). A coplanar system ($\phi = 0$) will stay coplanar forever. So, ϕ statistics must bear traces of formation history. One expects totally uncorrelated orbital spins ($\langle \phi \rangle = 90°$) for purely dynamical processes, and $\langle \phi \rangle \approx 0$ for a cascade fragmentation of a rotating protostellar cloud.

The study of ϕ statistics was made on the basis of the initial version of MSC (Tokovinin 1993). Three different sub-samples (systems with two visual orbits known, double-lined spectroscopic binaries in orbital visual systems, and distant tertiary components to orbital visual binaries) have shown a similar behavior: both extreme hypotheses (co-aligned and random orbital spins) can be rejected. The average ϕ is around 50°. The statistics is so poor and indirect that no additional information could be obtained. For example, a mixture of two populations with co-aligned and random orbital spins can fit the data, as well as a statistically uniform population with partially correlated orbit orientation.

The number of systems in each class may have increased now by a factor of ≈ 1.5, warranting a new study. However, a *significant* increase of the sample size is needed to probe the dependence of ϕ statistics on other factors (age, primary mass, environment, etc.). Efforts should be made to solve this problem either by direct interferometric resolution of close sub-systems or by precise astrometry to find the orientation of an inner orbit through small deviations in the motion of the wide pair.

3.4. Mass ratios

Important information on multiple star origin is contained in the mass ratio statistics. Unfortunately, the discovery bias is so large that this subject remains uncertain and controversial even for binaries. As for multiples, mass ratios in *catalogued* systems seem to correspond to random combinations of components drawn from the initial mass function (Tokovinin 1997a). Valtonen (1998) used the same material to argue that dynamical processes are important and they shift mass ratio towards 1, compared to a random selection. However, as long as bias is not taken into account, these results can not be considered as relevant to a *real* mass ratio statistics. In Fig. 4 the mass ratio at the outer hierachical level of triple and higher-multiplicity systems is plotted against orbital period.

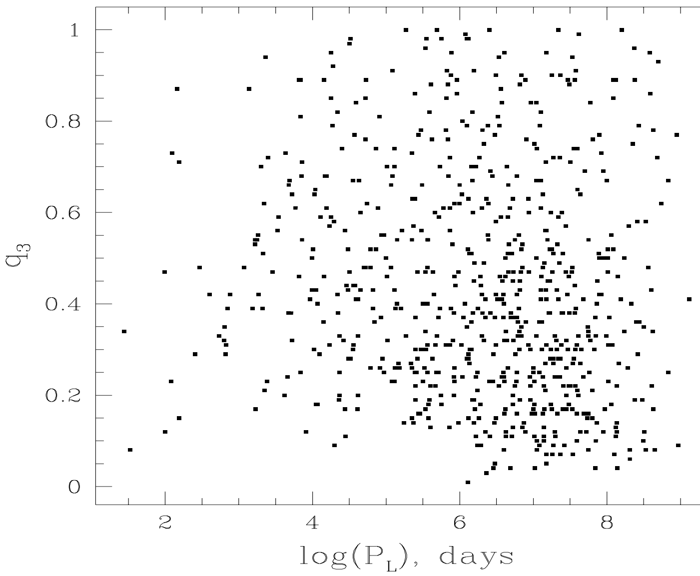


Figure 4. Mass ratio of the upper-level sub-systems q_3 (combined mass of the tertiary divided by combined mass of inner subsystem) as a function of their orbital periods P_L in days. When q_3 is larger than 1, its inverse value is plotted.

A tendency towards more uniform distribution at shorter periods is apparent, but it is not clear whether this not due to discovery bias.

Many short-period sub-systems have mass ratios very close to 1 (e.g. Aab in ADS 9731). A peak in the binary mass ratio distribution at $q = 1$ has been a subject of a long controversy. Re-examination of the data on double-lined spectroscopic binaries shows that the sharp peak at $q > 0.95$ is real for periods shorter that ≈ 30 days, but "twins" constitute only 10-20% of the total binary population at these periods and are not apparent in small samples (Tokovinin 2000). As a possible explanation it was suggested that twins have been formed by a strong accretion onto a binary which equalized component masses and at the same time shortened the period (Bate 2000).

3.5. Eccentricities

Eccentricities of multiple stars have not yet been an object of study. The highest known eccentricities, however, are found in inner sub-systems of multiple stars (Duquennoy et al. 1992; Tokovinin 1995). Dynamical interaction with outer orbit is evoked to explain this, the same mechanism acts on planetary orbits in wide binaries (Holman et al. 1997). Detection of a non-zero eccentricity in a close binary which would normally be tidally circularized may signal a presence of a yet undiscovered tertiary companion (Mazeh 1990).

An intriguing result is presented by Shatsky (2000) at this conference, namely that the wide outer sub-systems of multiple stars show a tendency towards circular orbits and do not follow the $f(e) = 2e$ distribution typical of wide binaries. This could be explained by the increased probability of a dynamical decay for multiples with eccentric outer orbits, due to the interaction of the distant component with inner pair(s). The transfer of angular momentum from the inner binary or its circumbinary disk to the tertiary also circularizes the outer orbit.

4. Relation of statistics to formation/evolution

4.1. Dissipative Kozai evolution

Modulation of the eccentricity of an inner binary in a multiple system has important consequences on the evolution of orbital parameters. This modulation, called Kozai cycles (Kozai 1962), is periodic. It can be strong for systems with initially almost perpendicular orbits, driving eccentricity very close to 1 (however, relativistic precession limits eccentricity growth, cf. Holman et al. (1997)). When this happens, components can no longer be treated as points, and their tidal interaction becomes important. As shown by Kiseleva & Eggleton (1998), in this case the Kozai cycle breaks, inner orbit becomes practically circular. This mechanism explains the formation of highly hierarchical triple systems like Algol from the systems with initially moderate P_L/P_S ratios. Systems with extreme eccentricities mentioned above could have been produced in the similar way, they are just too wide for tidal circularization to become effective.

In main-sequence dwarf binaries tidal force is a strong function of separation, and circularization becomes effective for periods shorter than $\approx$ 6 days (Zahn & Bouchet 1989). The periods of close sub-systems which passed through a dissipative Kozai evolution must then be below 6 days. So, inner sub-systems with initially longer periods and high mutual orbit inclination pass through this stage and have their periods shortened below 6^d. This consideration can explain the apparent over-abundance of 3-6 day periods compared to 6-12 day periods, as noted above. A statistical modeling of this process has not yet been made.

4.2. Independent multiplicity?

Looking at Fig. 3, one might think that multiple stars are just combinations of long- and short-period systems, taken randomly from some common distribution, and subject only to a stability constraint. In reality things are not so simple. Correlation of angular momenta, although not yet explained, speaks already against independent combination of close and wide sub-systems.

I made a cross-identification between a catalogue of spectroscopic binary orbits (Batten et al. 1989) and MSC, to see what fraction of SBs belong to multiple systems. To my astonishment, it turned out that this fraction strongly depends on the SB period, and is much higher for close binaries: 0.43 for $1-10$ day periods and only 0.08 for 10-100 day periods (Tokovinin 1997b). The difference is too high to be explained by observational selection (there is no selection here, close and wide sub-systems being discovered independently) or by the fact that for close SBs there remains a wider choice of stable tertiary orbits. Of the 5 close ($P < 10^d$) binaries in a DM91 sample 4 have tertiaries, and the remaining one is a suspected triple as well. By the way, many astronomers working on close binaries noted long time ago that the rate of discoveries of tertiary stars in these objects is very high.

Another confirmation of a statistical relation between close and wide binaries comes from my radial velocity survey of distant tertiary components of known binaries. The general results are not yet published, but the statistics of component discoveries in a more or less well-defined sub-sample is presented in Table 2. A fraction of spectroscopic binaries discovered is high: for a period range of $1 - 100$ days it is $(19 \pm 6)\%$, to be compared with 7.3% for the same period range in the DM91 sample.

Table 2. Spectroscopic binaries among distant tertiaries

Total number of components studied	59	100%
Constant radial velocity	35	59%
Variable radial velocity	11	19%
Spectroscopic orbits:	13	22%
$1 - 10$ days	7	12%
$10 - 100$ days	4	7%

So, the distant tertiaries have an increased proportion of close binaries, being somewhat "contaminated" by a binary syndrome, as compared to normal dwarfs. An explanation may be that the common environment which produced both tertiary and its previously known binary companion was favorable for close binary formation.

The same idea of common "binary-favorable" environment may be evoked to understand a correlation between close binaries and triples mentioned above. Alternatively, the formation of close sub-systems can be related to the presence of a tertiary in a more direct way, the latter being responsible for taking away the angular momentum from the inner pair and converting it into the present-day short-period system. One of the possible mechanisms for such interaction involves dissipative Kozai cycles. Another possibility is an interaction with tertiary at PMS stage which increases the accretion onto a binary and shrinks its orbit. An example of such young triple is found by Reipurth et al. (1999).

5. What should be done?

Although poor, the statistics of high-multiplicity systems does provide some new clues which call for theoretical explanation or modeling. The yet unexplained features include statistical relation of close binaries to higher multiplicity, partial correlation of orbital spin orientations, excess of inner sub-systems with $P_S < 6^d$, and the existence of a population of "twin" binaries with identical masses and $P < 30^d$.

From the observational side, much work remains to be done. The future efforts will, naturally, be directed towards obtaining multiplicity statistics in large volume-limited samples in a most complete way by combining precise radial velocities with high-resolution imaging. In a sample of 1000 G-type dwarfs, some

60 triple, 15 quadruple, and 4 quintuple systems are expected, illustrating the importance of a large sample size for meaningful multiple star statistics.

Even with modern techniques, discovery of multiple systems in a given sample will never be complete. Dynamical stability constraints will help to narrow down the parameter space of undiscovered components, so that for some objects the *true* multiplicity will be firmly established. But for remaining sample members the discovery biases will persist. It means that new methods still need to be developed to obtain reliable statistics from incomplete data sets.

It will be very important to probe high-multiplicity statistics in different populations, e.g. with respect to primary mass, age, formation environment. The first step will be done soon for low-mass PMS stars, where the number of discovered triples is perhaps already large enough for meaningful statistical analysis.

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