

Dynamics of Four Triple Systems

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4 ABSTRACT

5 Orbital motions in four hierarchical stellar systems discovered by speckle interferometry are studied.
6 Their inner orbits are relatively well constrained, while the long outer orbits are less certain. The
7 eccentric and misaligned inner orbits in the early-type hierarchies ϵ Cha (B9V, central star of the
8 5 Myr old association, $P = 6.4$ yr, $e = 0.73$), and I 385 (A0V, $P \sim 300$ yr, $e \sim 0.8$) suggest past
9 dynamical interactions. Their nearly equal masses could be explained by a dynamical decay of a 2+2
10 quadruple progenitor consisting of four similar stars. However, there is no evidence of the associated
11 recoil, so similar masses could be just a consequence of accretion from the same core. The other two
12 hierarchies, HIP 32475 (F0IV, inner period 12.2 yr) and HIP 42910 (K7V, inner period 6.8 yr), have
13 smaller masses and are double twins where both inner and outer mass ratios are close to one. A double
14 twin could either result from a merger of one inner pair in a 2+2 quadruple or can be formed by a
15 successive fragmentation followed by accretion.

16 *Keywords:* binaries:visual stars:multiple stars:individual

17 1. INTRODUCTION

18 Multiple stellar systems are very diverse, ranging from
19 compact planar worlds, where three or four stars are
20 tightly packed within 1 au, to wide systems of 0.1 pc
21 scale, often found in non-hierarchical configurations; see
22 Tokovinin (2021a) for a review. Hierarchies with separa-
23 tions of 1–100 au, in the middle of this range, are more
24 typical. Their dynamics (periods, eccentricities, mutual
25 orbit orientation) bears imprints of the formation pro-
26 cesses. However, only for a tiny fraction of known triple
27 systems the inner and outer orbits could be determined
28 or constrained owing to long (centuries and millenia)
29 outer periods and insufficient data. It is increasingly
30 clear that hierarchies were formed via several different
31 channels.

32 In this work, orbits are determined for four such sys-
33 tems (Table 1), continuing similar studies reported in
34 (Tokovinin 2021b; Tokovinin & Latham 2020; Tokovinin
35 2018a; Tokovinin & Latham 2017). Inner pairs in these
36 systems were discovered a decade ago by speckle interfer-
37 ometry, and the data accumulated to date allow calcula-
38 tion of the first inner orbits. The outer orbits are not yet
39 fully covered. Two systems (ϵ Cha and I 385) have simi-

40 lar components of early spectral type arranged in appar-
41 ently non-hierarchical configurations. Their inner orbits
42 have large eccentricities, suggesting that dynamical in-
43 teractions played a major role. The other two triples
44 contain solar-type stars and are double twins where a
45 pair of similar low-mass stars orbits the primary com-
46 ponent with mass comparable to the mass of the pair.
47 Despite apparent similarity, the two double twins have
48 very different dynamics: the first has quasi-circular and
49 aligned orbits, while in the other the inner orbit is highly
50 eccentric.

51 The input data and methods are briefly introduced in
52 Section 2. Sections 3–5 are devoted to individual sys-
53 tems. Their possible formation scenarios are discussed
54 in Section 6.

55 2. DATA AND METHODS

56 2.1. Speckle Interferometry

57 In the hierarchies studied here inner subsystems have
58 been discovered by speckle interferometry with the high-
59 resolution camera (HRCam) working on the 4 m tele-
60 scopes SOAR (Southern Astrophysical Research Tele-
61 scope) and Blanco located in Chile. HRCam, in
62 use since 2007, is based on the electron multiplica-
63 tion CCD detectors. The instrument, data processing,

Table 1. List of Multiple Systems

WDS	Name	HIP	HD	V	ϖ^a	P_{out}	P_{in}	Masses
(J2000)				(mag)	(mas)	(yr)	(yr)	(M_{\odot})
06467+0822	HDS 940 A,BC	32475	49015	7.04	13.75 G	80.3	12.2	1.40+(0.69+0.65)
08447-2126	HDS 1260 A,BC	42910	...	10.19	27.51 G	125	6.9	0.72+(0.37+0.36)
11596-7813	ϵ Cha	58484	104174	4.90	9.02 H	750:	6.4	(2.57+2.45)+2.54
17248-5913	I 385 AD,B	85216	157081	7.25	3.85 G	2000:	300:	(2.32+2.03)+1.99

^a Parallax codes: G — Gaia DR3 (Gaia Collaboration et al. 2021), H — Hipparcos (van Leeuwen 2007).

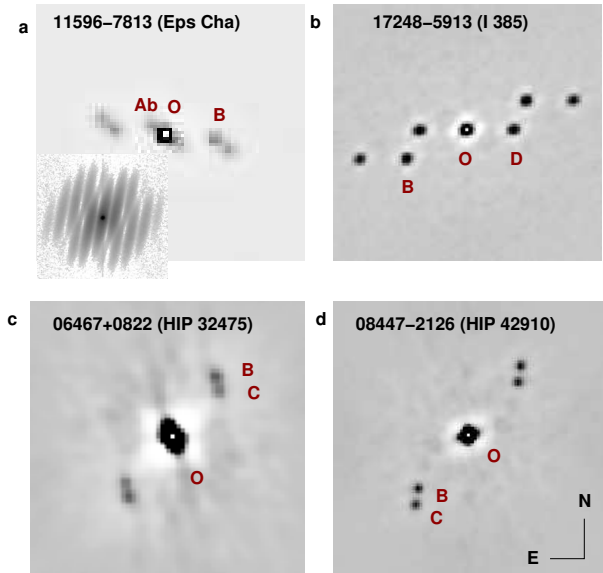


Figure 1. Speckle ACFs of triple stars recorded at SOAR (negative intensity rendering, standard orientation, arbitrary scale). In each panel, the white dot O marks the center, other labels indicate secondary peaks matching companion’s positions. (a) ϵ Cha on 2022.05, separations $0''.054$ and $0''.147$; insert shows the power spectrum. (b) I 385 on 2022.31, separations $0''.29$ and $0''.41$. (c) 06478+0822 on 2022.77, separations $0''.073$ and $0''.34$. (d) 08447-2126 on 2022.28, separations $0''.067$ and $0''.48$.

and performance are covered in (Tokovinin et al. 2010; Tokovinin 2018b). The latest series of measurements and references to prior observations can be found in (Tokovinin et al. 2022). Image cubes of 200×200 pixels and 400 frames are recorded mostly in the y (543/22 nm) and I (824/170 nm) filters with an exposure time of 25 ms or shorter and a pixel scale of 15 mas. In the y filter, the diffraction-limited resolution of 30 mas can be attained, and even closer separations can be measured via careful data modeling. On the other hand, the I filter offers deeper magnitude limit and better sensitivity to faint, red companions.

Image cubes are processed by the standard speckle method based on calculation of the spatial power spec-

trum and image auto-correlation function (ACF) derived from the latter. The 180° ambiguity of position angles inherent to this method is resolved by examination of the shift-and-add (“lucky”) images and by comparison with prior data. In a triple star, the angles of subsystems are related, so the better-defined orientation of the outer pair constrains the orientation of the inner subsystem. Figure 1 illustrates speckle data on the triple systems studied here. Recall that the positions and relative photometry are determined by modeling the power spectra, not by fitting the ACF peaks.

2.2. Orbit calculation

As in the previous papers, an IDL code that fits simultaneously inner and outer orbits in a triple system has been used (Tokovinin 2017).¹ The method is presented in Tokovinin & Latham (2017). No useful radial velocity measurements are available for the systems studied here, so only positional measurements are used. The weights are inversely proportional to the squares of adopted measurement errors which range from 2 mas to $0''.05$ and more (see Tokovinin 2021b, for further discussion of weighting).

Motion in a triple system can be described by two Keplerian orbits only approximately, but the effects of mutual dynamics are too small to be detectable with the current data. The code fits 14 elements of both orbits and the additional parameter f – the wobble factor, ratio of the astrometric wobble axis to the full axis of the inner orbit. For resolved triples, $f = q/(1+q)$, where q is the inner mass ratio. When the inner subsystem is not resolved, measurements of the outer pair refer to the photo-center of the inner pair, and the wobble amplitude corresponds to a smaller factor $f^* = q/(1+q) - r/(1+r)$, where r is the flux ratio. The code `orbit4.pro` can accept a mixture of resolved and unresolved outer positions; it adopts a fixed ratio f^*/f , specified for each system as input parameter.

¹ Codebase: <http://dx.doi.org/10.5281/zenodo.321854>

For the two early-type outer pairs discovered visually, position measurements at SOAR are complemented by the historic micrometer and speckle data retrieved from the Washington Double Star (WDS) database (Mason et al. 2001) on my request. Although such data extend the time coverage to almost 200 yr (for ϵ Cha), it is still too short for constraining outer periods of several centuries. To avoid the degeneracy of outer orbits, some elements are fixed to reasonable values that agree with the estimated masses. The resulting outer orbits are only representative; however, they are still useful for the assessment of mutual dynamics.

The elements of inner and outer orbits in the selected triple systems are given in Table 3 in standard notation. Considering the uncertain nature of outer orbits, the formal errors of their elements are meaningless, so they are not provided. Individual positions and their residuals to orbits are listed in Table 2, available in full electronically. Compared to the published HRCam data, the positions are corrected for the small systematics determined in (Tokovinin et al. 2022) and, in a few cases, reprocessed. The second column indicates the subsystem; for example, A,BC refers to the angle and separation between A and unresolved pair BC, while A,B refers to the position of resolved component B relative to A.

3. EPSILON CHAMAELEONTIS

The bright ($V = 4.90$, $K = 4.98$ mag) B9V star ϵ Cha (HR 4583, HD 104174, HIP 58484, WDS J11596–7813) is the central star of the young (5 ± 2 Myr, Dickson-Vandervelde et al. 2021) ϵ Cha association located at ~ 100 pc average distance (Murphy et al. 2013). The star has been resolved in 1835.93 into a $1''.6$ binary with comparably bright components by Herschel (1847), designated as HJ 4486. Subsequent monitoring with visual micrometers revealed a slowly decreasing separation with little change in position angle. Speckle-interferometric and Hipparcos measurements in the 1990s documented a separation of $\sim 0''.4$.

Considering that the pair A,B was closing down and lacked recent measurements, it was observed at SOAR in 2015.25 on request by Ross Gould (Tokovinin et al. 2016). Quite unexpectedly, ϵ Cha was revealed as a tight triple consisting of similar stars (Figure 1a). The inner pair Aa,Ab with a separation of 51 mas was expected to have a short orbital period and, indeed, its fast orbital motion was detected in the following years (Briceño & Tokovinin 2017). In 2022 Aa,Ab has com-

pleted one full revolution since its discovery, and its orbit with a period of 6.4 yr is determined here.

The fluxes of the three components of ϵ Cha are similar, but not exactly equal, which helps to establish the orientation. As shown in Figure 1a, the ACF peak below B is slightly weaker than the peak of B itself, thus defining the orientation of Ab relative to Aa as indicated. The 13 SOAR measurements in the y filter average to $\Delta y_{Aa,Ab} = 0.25$ mag and have an rms scatter of 0.04 mag. At the same time, $\Delta y_{Aa,B} = 0.11$ mag with a scatter of 0.07 mag. The combined magnitude of $V = 4.90$ mag leads to the individual V magnitudes of Aa, Ab, and B: 5.98, 6.23, and 6.09 mag, respectively.

Speckle interferometry allows a position angle change by 180° (flip), but only simultaneous flips of both pairs in a triple are allowed. The orientation of A,B is defined by the historic measurements, thus fixing the angle of Aa,Ab. However, when in 2019 Aa,Ab closed down below the diffraction limit, the ACF peaks overlapped and it was no longer possible to discriminate reliably between opposite angles of the inner pair. The data of 2019 were originally processed under the assumption that Ab is located to the north of Aa, extrapolating its retrograde motion from the previous years. However, a negative $\Delta y_{Aa,Ab}$ indicated that this assumption was incorrect, as also confirmed by the orbit. The SOAR observations in 2019 were re-fitted with the reversed orientation of Aa,Ab, which also affected the measured positions of Aa,B.

The orbits of Aa,Ab and A,B were fitted jointly. Apart from the WDS data, one speckle measurement made at Gemini-S in 2017.4 is used (Horch et al. 2019), the rest are SOAR measurements. The resulting wobble factor $f = 0.48 \pm 0.02$ indicates that the masses of Aa and Ab are equal, $q_{Aa,Ab} = 0.92 \pm 0.08$. The first attempt to compute the orbit of Aa,Ab using wrong quadrants in 2019 resulted in an unrealistically small mass sum, but after quadrant correction the orbit of Aa,Ab becomes almost perfect (weighted rms residuals 0.9 mas) and corresponds to the inner mass sum of $5.2 \pm 0.6 M_\odot$ using the Hipparcos parallax of 9.02 mas. The outer orbit, however, is not yet constrained by the observed arc, allowing a wide range of solutions. Two orbits of A,B are listed in Table 3: a circular one with $P = 751$ yr and an eccentric orbit with $P = 460$ yr. The circular orbit is adopted below; it corresponds to the mass sum of $7.9 M_\odot$. Dynamical stability of the triple system requires a separation of $> 0''.2$ at the outer periastron, so the outer eccentricity should not exceed 0.8.

Note that the inner pair moves clockwise, the outer pair counterclockwise, so the two orbits cannot be coplanar. However, without identification of the correct as-

Table 2. Positional Measurements and Residuals

WDS	System	T	θ	ρ	σ	O- C_θ	O- C_ρ	Ref. ^a
		(yr)	($^\circ$)	($''$)	($''$)	($^\circ$)	($''$)	
06467+0822	B,C	2015.9063	2.6	0.0823	0.005	-1.0	0.0037	S
06467+0822	B,C	2015.9063	1.5	0.0835	0.005	-2.1	0.0049	S
06467+0822	B,C	2016.9575	33.5	0.0812	0.005	3.8	-0.0057	S

^a H: Hipparcos; M: visual micrometer measurement; S: speckle interferometry at SOAR; s: speckle interferometry at other telescopes.

(This table is available in its entirety in machine-readable form)

Table 3. Orbital Elements

WDS	System	P	T	e	a	Ω	ω	i	f
		(yr)	(yr)		($''$)	($^\circ$)	($^\circ$)	($^\circ$)	
06467+0822	B,C	12.20	2011.70	0.095	0.0827	54.7	171.8	32.3	-0.50
		± 0.38	± 0.06	± 0.032	± 0.0033	± 8.0	± 20.5	± 5.3	± 0.03
06467+0822	A,BC	80.3	2029.0	0.072	0.382	23.6	324.7	30.7	...
		± 1.9	± 5.7	± 0.055	± 0.108	± 7.5	± 20.0	± 5.7	...
08447-2126	B,C	6.845	2016.416	0.948	0.0889	8.6	0	160.0	-0.48
		± 0.034	± 0.023	± 0.006	± 0.009	± 0.6	fixed	fixed	± 0.03
08447-2126	A,BC	125	2020.05	0.292	0.783	26.3	235.0	160.0	...
11596-7813	Aa,Ab	6.43	2018.57	0.733	0.0541	21.1	116.2	111.2	0.48
		± 0.09	± 0.06	± 0.020	± 0.0022	± 1.5	± 2.2	± 1.8	± 0.02
11596-7813	A,B	751	1837.4	0.0	1.481	181.4	0	83.5	...
11596-7813	A,B	460	2061.3	0.75	1.092	173.8	213.6	78.2	...
17248-5913	A,D	300	1969.0	0.80	0.280	271.5	242.0	90.0	0.43
17248-5913	AD,B	2000	1566	0.12	1.13	243.6	12.1	111.0	...

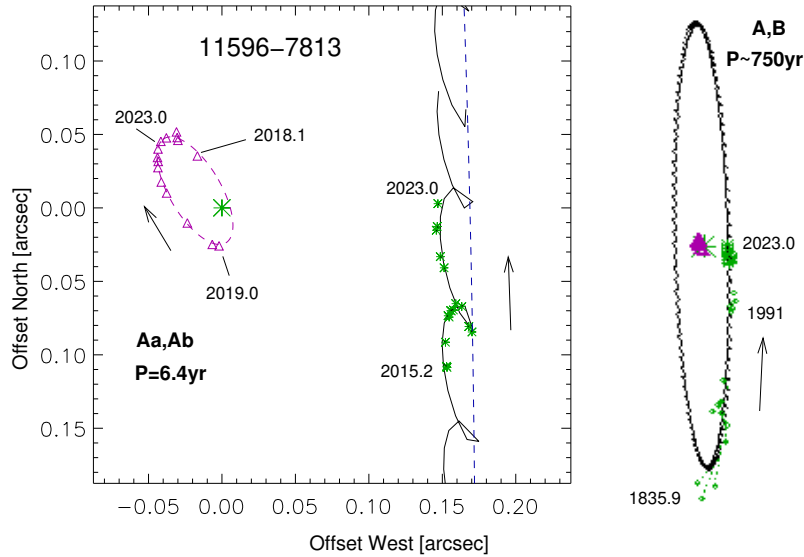


Figure 2. Orbits of ϵ Cha. The right-hand plot shows the full circular outer orbit (crosses denote the less accurate micrometer measurements, squares show the resolved speckle data). The left-hand plot shows SOAR measurements of the inner pair (magenta ellipse and triangles) and the wavy line of the Aa,B motion with the superimposed wobble. The blue dashed line shows outer orbit without wobble.

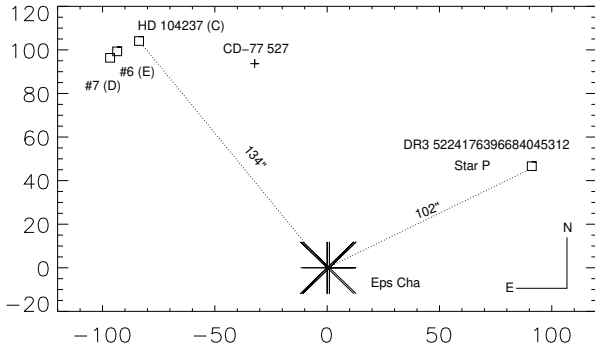


Figure 3. Closest neighbors of ϵ Cha (axis scale in arcseconds) in Gaia DR3.

Table 4. Neighbors of ϵ Cha in Gaia DR3

Name	Sep.	G	ϖ	μ_{α}^*	μ_{δ}	RUWE
	(arcsec)	(mag)	(mas)	(mas yr $^{-1}$)		
ϵ Cha (AB)	0	4.78	9.02:	-42.9	-10.1	...
Star P	101.9	15.23	9.413	-39.0	-5.5	1.1
HD 104237 (C)	133.9	6.56	9.380	-39.3	-5.8	2.1
Eps Cha#6 (E)	136.7	13.01	9.769	-38.7	-3.1	1.4
Eps Cha#7 (D)	136.8	12.78	9.980	-42.8	-4.2	1.6

214 cending nodes of both orbits, the mutual inclination can
 215 take two possible values, 156° or 34° for the circular
 216 outer orbit (156° and 41° for the eccentric one). The
 217 first value corresponds to counter-aligned orbital angu-
 218 lar momenta, while the second implies only a modest
 219 inclination. It is likely that mutual inclination and inner
 220 eccentricity vary in Lidov-Kozai cycles. The large inner
 221 (and possibly outer) eccentricities attest, indirectly, to
 222 dynamical interaction between the orbits.

223 The Multiple Star Catalog (Tokovinin 2018c) and
 224 the WDS associate ϵ Cha with another multiple sys-
 225 tem, HD 104237 (HIP 58520, DX Cha, $V = 6.60$ mag,
 226 A7Ve) located at an angular distance of $134''$ (projected
 227 separation 15 kau or 0.07 pc; FGL 1 AB,C). The pro-
 228 jected separation implies an orbital period of ~ 0.5 Myr
 229 if these stars are gravitationally bound. HD 104237
 230 is a spectroscopic binary with a period of 19.86 days
 231 that has been extensively studied; it is accreting from
 232 a circumbinary disk (Dunhill et al. 2015). Furthermore,
 233 HD 104237 is surrounded by a swarm of five faint low-
 234 mass stars within $15''$ according to Grady et al. (2004)
 235 and Gaia; the WDS code of this system is J12001-7812.
 236 I looked for objects within $3'$ radius of ϵ Cha in Gaia
 237 DR3 (Gaia Collaboration et al. 2021) and found an-
 238 other association member, DR3 5224176396684045312
 239 ($G = 15.23$ mag, parallax 9.413 ± 0.025 mas, proper mo-
 240 tion $(-38.95, -5.48)$ mas yr $^{-1}$) at a distance of $101''$,
 241 denoted provisionally as star P. Location of the neigh-
 242 bors on the sky is illustrated in Figure 3. The star
 243 CD-77 527 situated between ϵ Cha and HD 104237 does
 244 not belong to the association (parallax 3.44 mas).

245 Gaia does not provide parallax of ϵ Cha, while
 246 Hipparcos measured 9.02 ± 0.36 mas (new reduction,
 247 van Leeuwen 2007, 8.95 ± 0.58 mas in the original cat-
 248 alog). Comparison of the Gaia and Hipparcos posi-
 249 tions gives the best estimate of the proper motion (PM),
 250 $(-42.85, -10.14)$ mas yr $^{-1}$. Orbital motion of B rela-
 251 tive to A with a speed of 12.8 mas yr $^{-1}$ is directed to

252 the north. However, for multiples with equal compo-
 253 nents the centers of mass and light coincide ($f^* = 0$),
 254 so correction of the PM for the orbital motion is not
 255 needed. Gaia DR3 measured a parallax of 9.38 ± 0.05
 256 for HD 104237, compatible within errors with the Hip-
 257 parcos parallax of ϵ Cha. The formal errors of Gaia and
 258 Hipparcos parallaxes cannot be fully trusted because as-
 259 trometry of unresolved multiple systems is often biased.
 260 Table 4 lists parallaxes and PMs of the neighbors found
 261 in Gaia. Capital letters correspond to the components'
 262 designations in the WDS and MSC. The last column
 263 gives the Reduced Unit Weight Error (RUWE) as an
 264 indicator of the Gaia astrometric quality and potential
 265 subsystems. The closest satellite of HD 104237 at $1''4$
 266 separation (GRY 1 AF) has no parallax and PM in Gaia
 267 DR3, but the stability of its relative position over time
 268 proves that it is bound.

269 The projected separations of ϵ Cha to its neighbors
 270 are within 15 kau, typical for wide binaries and triples
 271 and suggesting that they may be bound. However, the
 272 PM differences of ~ 5 mas yr $^{-1}$ (2.5 km s $^{-1}$) in Table 4
 273 appear highly significant. Note also that two satellites
 274 of HD 104237, ϵ Cha #6 (E) and #7 (D) at $10''$ and
 275 $15''$ separations, respectively, have measurably different
 276 parallaxes, implying that this pair might be ~ 7 pc closer
 277 to the Sun and simply projects onto HD 104237. So, the
 278 status of the neighbors remains undetermined. They
 279 could be either just independent members of the asso-
 280 ciation or members of a bound (but likely dynamically
 281 unstable) stellar system.

282 The relative photometry of the ϵ Cha components
 283 allows to place them on the color-magnitude diagram
 284 (CMD). The individual colors are not measured, but,
 285 given similar magnitudes, they should be close to the
 286 combined color $V - K = -0.08$ mag. In Figure 4,
 287 the colors are arbitrarily offset from this value for il-
 288 lustration. Overall, the Hipparcos distance, inner orbit,
 289 and isochrone lead to consistent masses around $2.5 M_{\odot}$.
 290 However, the isochrone is not monotonous in this region,

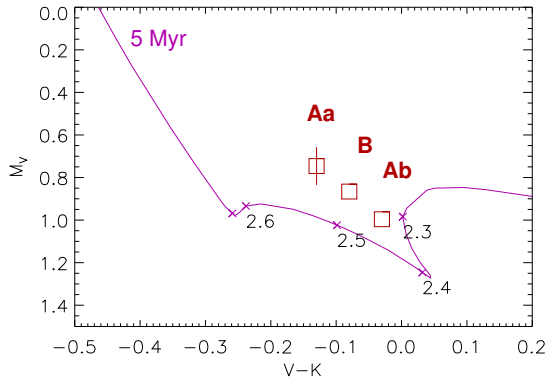


Figure 4. Location of ϵ Cha components Aa, B, and Ab (red squares) on the $(V, V - K)$ CMD. The error bar indicates the distance modulus uncertainty of ± 0.08 mag. The magenta curve is a 5 Myr PARSEC isochrone for solar metallicity (Bressan et al. 2012) with masses marked by asterisks and numbers.

291 which is sometimes called H-peak (Guo et al. 2021) and
 292 corresponds to the ignition of hydrogen burning in young
 293 stars. At 5 Myr age, the H-peak is located at M_G be-
 294 tween 0 and 1 mag, matching the absolute magnitudes
 295 of ϵ Cha components. Given the uncertainties in the dis-
 296 tance and color, potential inaccuracy of the isochrone,
 297 and its particular shape, it is hazardous to infer masses
 298 from the isochrone; the masses listed in Table 1 are ten-
 299 tentative.

300 Considering the young age of ϵ Cha and the contin-
 301 ued accretion on its neighbor HD 4104237, it was worth
 302 checking for the presence of hydrogen emissions in the
 303 spectrum. An optical echelle spectrum of ϵ Cha has been
 304 taken on 2022 February 25 with the CHIRON echelle
 305 spectrometer on the 1.5-m telescope at Cerro Tololo
 306 (Tokovinin et al. 2013). The wide and deep hydrogen
 307 Balmer lines have no signs of emission, as established
 308 previously by Lyo et al. (2008). Apart from that, the
 309 spectrum is almost featureless. One notes only sharp
 310 telluric absorptions in the red part and a few very shal-
 311 low and wide stellar lines. Thus, any residual gas around
 312 ϵ Cha has been expelled and this system is not accreting
 313 at present. Its potential formation scenario is discussed
 314 below in Section 6.

315 4. INNES 385

316 This remarkable quadruple stellar system is known as
 317 HIP 85216, HD 157081, WDS J17248–5913, and I 385.
 318 The bright visual triple consisting of the $0''.5$ pair A,B
 319 with companion C at $17''$ separation has been discovered
 320 by R. Innes in 1901 (Innes 1905); star C has similar PM
 321 and parallax, hence it belongs to the system. Another
 322 star E listed in the WDS ($28''.01$, $109^\circ 3$, $G = 13.50$ mag)
 323 is optical, as evidenced by its distinct Gaia parallax (0.47

324 mas) and PM of $(-0.4, -6.3)$ mas yr $^{-1}$. The inner com-
 325 panion D, similar in brightness to A and B, has been
 326 discovered in 2008.5 by speckle interferometry (WSI 87
 327 AD) at $0''.26$ separation, while A,B was at $0''.39$, in
 328 a spectacular triangular configuration (Tokovinin et al.
 329 2010) shown in Figure 1b. The object was regularly
 330 visited by the SOAR speckle camera since its discov-
 331 ery. During 14 years (2008.5 – 2022.3) the A,D pair
 332 has opened up slightly (from $0''.26$ to $0''.28$) at a rate
 333 of 2 mas yr $^{-1}$ with constant position angle, while A,B
 334 moved faster. A preliminary analysis of this system was
 335 provided in Tokovinin et al. (2016).

336 Examination of all available data has led to the
 337 firm conclusion that the speckle-interferometric obser-
 338 vation of this star by the CHARA group in 1990.35
 339 (Hartkopf et al. 1993) actually resolved the triple. Brian
 340 Mason consulted the archive and, indeed, the system
 341 was noted as having “possible third component”. The
 342 position of A,D was measured at $270^\circ 2$ and $0''.1994$. This
 343 pre-discovery observation has not been published at the
 344 time, awaiting for a confirmation; it is used here. Cu-
 345 riously, the CHARA team also observed ϵ Cha at a 4
 346 m telescope in the 1990s three times, but they have not
 347 discovered the subsystem Aa,Ab.

348 Hipparcos measured the parallax of A as 3.15 ± 0.96
 349 mas (van Leeuwen 2007). Gaia does not give parallax
 350 of A because it is not a point source. However, the ac-
 351 curate Gaia DR3 parallax of star C (3.848 ± 0.013 mas)
 352 fixes the distance to this system. The Gaia astromet-
 353 ry of C is of good quality (RUWE=0.98). The PM of
 354 C, $(-8.989, -11.557)$ mas yr $^{-1}$, matches the Hipparcos
 355 PM of A, $(-8.7, -14.5)$ mas yr $^{-1}$; however, the latter
 356 is a blend of A, B, and D biased by motion in the inner
 357 triple. The PM of A derived from its Hipparcos and
 358 Gaia positions is $(-5.39, -12.26)$ mas yr $^{-1}$.

359 The median magnitude differences of A with B and
 360 D in the y band are 0.49 and 0.42 mag, respectively (D
 361 is slightly brighter than B). Considering the combined
 362 magnitude $V = 7.25$ mag, the individual V magnitudes
 363 of A, B, and D are 8.16, 8.65, 8.58 mag, respectively.
 364 The absolute magnitudes match main-sequence stars of
 365 masses 2.32, 1.99, and 2.03 M_\odot , and the combined spec-
 366 tral type A0V corresponds to a star of 2.3 M_\odot .

367 The fact that the inner, closer pair A,D moves slower
 368 than the wider pair A,B is unusual. Tokovinin et al.
 369 (2016) proposed two explanations. Star D could move
 370 on a wide orbit around A,B and project onto it. This
 371 configuration has a low probability and, moreover, the
 372 wide A,D orbit could be dynamically unstable with re-
 373 spect to the outer companion C. The other explanation
 374 of apparently slow A,D motion is because it is near apas-
 375 tron of an eccentric and highly inclined orbit. This more

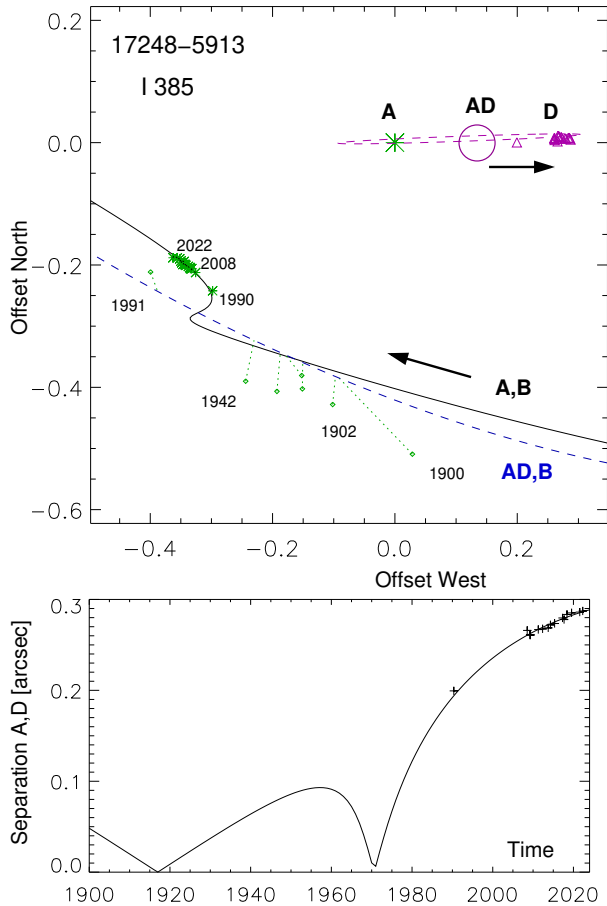


Figure 5. Orbital motion of the inner triple I 385. Top: motion of B relative to A (full line and green asterisks) or relative to the AD photo-center (blue dashed line and small diamonds). The magenta line and triangles show the orbit of A,D on the same scale. Star A is placed at the coordinate origin. The plot on the bottom shows angular separations of A,D vs. time. The inner pair was closer than $0''.1$ throughout most of the 20th century and for this reason it has been missed by visual observers.

376 natural hypothesis is adopted and further explored here.
 377 The observations do not cover the long orbital periods of
 378 AD,B and A,D; the short observed segments can match
 379 a wide range of possible orbits. The question is whether
 380 some of those potential orbits are compatible with the
 381 distance and estimated masses. To answer it, just a pair
 382 of plausible orbits suffice.

383 First, I studied the motions of AD,B and A,D separa-
 384 tely. A crude orbit of AD,B with $P = 1244$ yr was
 385 suggested in (Tokovinin et al. 2016). I assume that the
 386 historic micrometer measurements of AD,B refer to the
 387 unresolved inner pair AD. The resolved speckle measure-
 388 ments of A,B and A,D were transformed by replacing A
 389 with the average positions of A and D (center of mass),
 390 assuming that A and D are equal. After the initial fit,

391 the elements P and a were fixed to the values that match
 392 the expected mass sum of $6.3 M_{\odot}$. The eccentricity
 393 of AD,B, essentially unconstrained, is fixed to a small
 394 value (a large eccentricity would render the inner pair
 395 dynamically unstable). The actual values of P, a can be
 396 substantially larger than those adopted here.

397 For the inner orbit of A,D, I adopted the period of
 398 300 yr estimated from the projected separation, fixed
 399 $e = 0.8$ and $i = 90^{\circ}$, and selected the element ω to ob-
 400 tain the target mass sum of $4.3 M_{\odot}$. The resulting orbit
 401 fits well the observed slow motion of A,D. At present, the
 402 rate of its opening up decreases, and in several decades
 403 the pair will start to close down. In the final iteration,
 404 I used the `orbit4.pro` code to model both orbits si-
 405 multaneously (see Table 3). The masses quoted above
 406 correspond to the wobble factor $f = 0.47$, while the fit-
 407 ted value is 0.43 ± 0.13 . With the parallax of 3.85 mas,
 408 the inner and outer mass sums are 4.4 and $6.3 M_{\odot}$ and,
 409 by design, match the photometric mass sums.

410 The orbits are illustrated in Figure 5. One notes that
 411 the first measurement of A,B by Innes in 1900 is in-
 412 accurate. Five micrometer measurements of AD,B in
 413 1963–1979 are omitted, as well as the highly discrepant
 414 measurement by Innes in 1909.6 (discrepant micrometer
 415 measurements are common). The speckle measurement
 416 in 1990.35 by Hartkopf et al. (1993) at $0''.384$ separation
 417 matches the resolved position of A,B rather than AD,B
 418 (indeed, the triple was resolved at the time but not an-
 419 nounced), while the Hipparcos position in 1991.25 at
 420 $0''.452$ better fits the unresolved pair AD,B; it was likely
 421 biased by the triple nature of the source. The tentative
 422 orbits demonstrate that the slow motion of A,D is com-
 423 patible with an edge-on eccentric orbit. This orbit also
 424 explains why the triple has not been discovered earlier:
 425 throughout most of the 20th century A,D remained too
 426 close for a visual resolution.

5. DOUBLE TWINS HIP 32475 AND HIP 42910

427
 428 The two triple systems featured in this Section have
 429 some common features. Both are double twins where a
 430 more massive primary star A is orbited by a twin sec-
 431 ondary pair of low-mass stars B and C. The magnitude
 432 difference of B and C relative to A is substantial, about 3
 433 mag, as in other similar double twins (Tokovinin 2018a).
 434 Yet another similarity are moderate ratios of the outer
 435 and inner periods.

436 The outer pairs in these two systems were discovered
 437 by Hipparcos and are named HDS 940 and HDS 1260,
 438 respectively, in the WDS. Both secondaries were re-
 439 solved into close pairs at SOAR in 2014 and 2015. Inde-
 440 pendently, Horch et al. (2017) discovered the triple na-
 441 ture of HIP 42910 in 2012. This team also observed

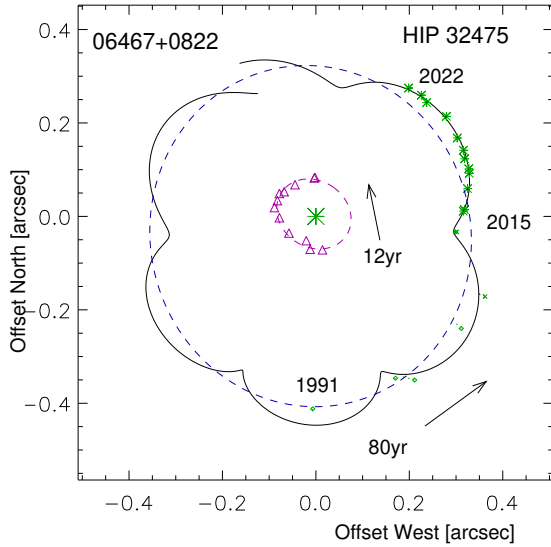


Figure 6. Orbits of HIP 32475 with periods of 12.2 yr and 80.3 yr. The blue dashed line marks the outer orbit without wobble that describes motion of BC around A, the black solid line is the motion of B relative to A. The orbit of B,C is plotted around coordinate origin on the same scale by the magenta line and triangles.

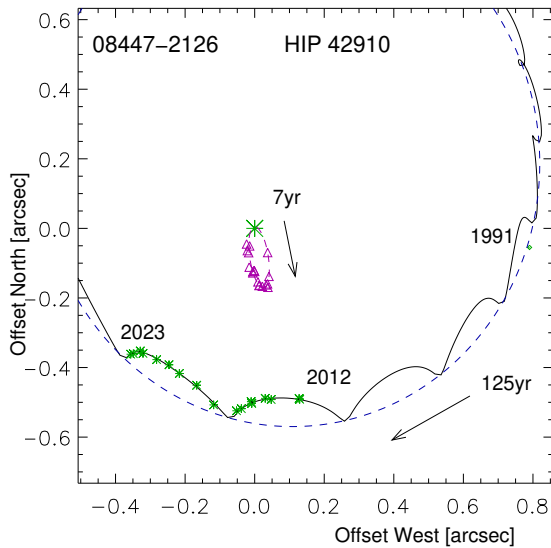


Figure 7. Orbits of HIP 42910; periods 6.9 yr and 125 yr.

442 HIP 32475 at the 3.5 m WIYN telescope five times from
 443 1998 to 2012. They published only measurements of the
 444 outer pair A,BC and apparently missed the subsystem.
 445 Figure 1 shows typical speckle ACFs of these triples in
 446 the I band. Both systems are not resolved by Gaia. Or-
 447 bital motion causes an increased astrometric noise and
 448 potentially affects parallaxes, although the bias caused
 449 by the century-long outer orbits might be small.

450 The orbits are plotted in Figures 6 and 7 and their
 451 elements are given in Table 3. The positional mea-

452 surements come from Hipparcos (outer pairs, epoch
 453 1991.25), publications by Horch et al. (e.g. Horch et al.
 454 2017), and SOAR. The coverage of both inner orbits is
 455 adequate, but the outer arcs are covered only partially.
 456 The shorter 80 yr outer orbit of HIP 32475 was deter-
 457 mined by free fit, but for HIP 42910 the outer period
 458 and inclination were fixed. Preliminary orbits for this
 459 triple with periods of 106 and 9.06 yr were published
 460 by Horch et al. (2021); they disagree with all measure-
 461 ments available at present. A preliminary outer orbit
 462 of HIP 32475 with $P = 128.9$ yr has been computed by
 463 Cvetković & Pavlović (2020).

464 The magnitude difference between components B and
 465 C of HIP 42910 is close to zero, so they can be swapped.
 466 An alternative to the eccentric inner orbit with $P = 6.8$
 467 yr could be a highly inclined near-circular orbit with ap-
 468 proximately double period. A quasi-circular orbit was
 469 fitted to the measurements of B,C with suitably changed
 470 quadrants ($P = 15.4$ yr, $a = 0''.185$, $e = 0.22$). How-
 471 ever, its agreement with the measurements is worse, and the
 472 inner mass sum of $1.28 M_{\odot}$ is much larger than allowed
 473 by the absolute magnitudes. So, the eccentric orbit of
 474 HIP 42910 B,C is the correct choice. However, the lack
 475 of measurements near its periastron, when the subsys-
 476 tem is below the SOAR resolution limit, does not fully
 477 constrain all elements. For this reason I fixed the in-
 478 ner elements ω and i to the values that agree well with
 479 the data and lead to the expected inner mass sum of
 480 $0.73 M_{\odot}$ (the free fit gives a slightly larger mass sum).
 481 The next inner periastron will occur in 2023.25, and the
 482 latest observation in 2023.0 confirms the decreasing sep-
 483 aration.

484 Speckle interferometry at SOAR gives reliable mea-
 485 surements of the magnitude differences in the spec-
 486 tral band close to I . These data are assembled in
 487 Table 5. For HIP 32475, the combined I magnitude
 488 should be close to the G -band magnitude, 6.95 mag (the
 489 color indices are moderate). This assumption and the
 490 isochrones agree with the measured combined V and K
 491 magnitudes. For the redder star HIP 42910, I adopt the
 492 combined $I = 8.70$ mag based on the following argu-
 493 ment. After splitting the flux between components and
 494 deriving their absolute I magnitudes, I use the PAR-
 495 SEC isochrone (Bressan et al. 2012) for 1 Gyr and solar
 496 metallicity to estimate the masses and the combined V
 497 and K magnitudes of the system (10.10 and 7.10 mag,
 498 respectively). They are compared to the actually mea-
 499 sured magnitudes (10.19 and 7.00 mag), and the best
 500 agreement is reached with the adopted combined I .

Table 5. Photometry and Masses of HIP 32475 and 42910

HIP	Parameter	A+B+C	A-B	B-C	A	B	C
32475	I (mag)	6.95	3.71	0.32	7.02	10.59	10.91
	M (M_{\odot})	1.40	0.69	0.65
42910	I (mag)	8.70	2.97	0.06	8.83	11.80	11.86
	M (M_{\odot})	0.72	0.37	0.36

501 In HIP 32475, the main star A, of F0IV spectral type,
 502 is a γ Dor pulsating variable V830 Mon. Its photometri-
 503 cally estimated mass, $1.40 M_{\odot}$, is close to the estimated
 504 mass sum of the inner pair, $1.34 M_{\odot}$, so both inner and
 505 outer mass ratios are close to one (a double twin); the
 506 inner orbit and parallax give the mass sum of $1.44 M_{\odot}$.
 507 The inner mass ratio is directly measured by the wob-
 508 ble factor and, within errors, matches the photometric
 509 masses. The outer mass ratio can be checked by compar-
 510 ing the outer orbital motion with the proper motion
 511 anomaly, PMA (Brandt 2018). It equals $(-10.7, -5.4)$
 512 mas yr^{-1} for the Gaia DR2 epoch of 2015.5, while the
 513 outer orbit predicts an effective motion of $(+20.6, +11.1)$
 514 mas yr^{-1} . The ratio of PMA to orbital motion is close
 515 to -0.5 and implies $q_{A,BC} \approx 1$ if the photo-center motion
 516 is attributed to star A and the light of BC is neglected.

517 The estimated mass of HIP 42910 A, $0.71 M_{\odot}$, ap-
 518 proximately matches its spectral type K7V. This is also
 519 a double twin. The inner orbit was tuned to obtain the
 520 expected mass sum of $0.73 M_{\odot}$, as noted above. The
 521 poorly constrained outer orbit yields a mass sum of 1.45
 522 M_{\odot} .

523 The similarity of those two hierarchies in terms of
 524 mass ratios and periods contrasts with very differ-
 525 ent character of their orbital motions. The orbits in
 526 HIP 32475 have moderate eccentricities and are oriented
 527 almost face-on. The most likely value of the mutual in-
 528 clination is $18^{\circ} \pm 6^{\circ}$ (the alternative value of 60° would
 529 have caused Lidov-Kozai cycles that would increase the
 530 inner eccentricity). The period ratio is small, 6.6, al-
 531 though it is not yet accurately known. Dynamical inter-
 532 actions in this low-hierarchy system are expected to be
 533 strong, and a mean motion resonance is possible. This
 534 triple system belongs to the family of “dancing twins”
 535 (Tokovinin 2018a). On the other hand, in HIP 42910
 536 the inner orbit has a large eccentricity of 0.95, and the
 537 mutual inclination is either 6° or 90° .

538 6. DISCUSSION

539 It is firmly established that masses of stars in bina-
 540 raries are correlated instead of being chosen randomly
 541 (Duchêne & Kraus 2013; Moe & Di Stefano 2017). This
 542 trend extends to hierarchical systems. Quadruplets

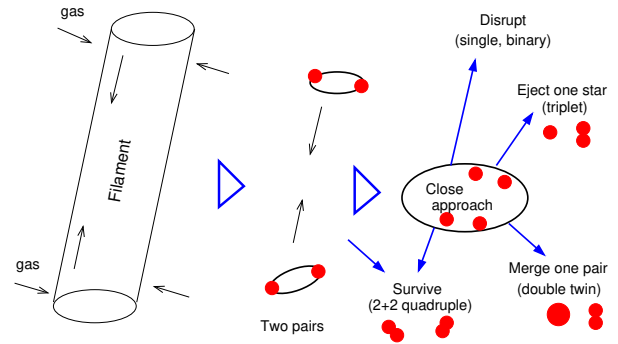


Figure 8. Possible scenario of forming hierarchical systems with comparable-mass components: triplets, quadruplets, and double twins.

543 consisting of four similar stars arranged in 2+2 hier-
 544 archy stand apart as a distinct family of ϵ Lyr type,
 545 although their orbital separations span a wide range
 546 (Tokovinin 2021a). Similarity of masses is naturally ex-
 547 plained by gas accretion onto a binary that tends to
 548 equalize masses and at the same time shrinks the or-
 549 bits (Tokovinin & Moe 2020). Extension of this idea to
 550 triples helps to explain double twins where both inner
 551 and outer mass ratios are close to one (Tokovinin 2018a).
 552 However, existence of hierarchical systems of three sim-
 553 ilar stars (triplets) like ϵ Cha with an outer mass ratio
 554 of 0.5 challenges the accretion scenario.

555 A possible path to form triplets is via dynamical decay
 556 of a 2+2 quadruple system. This scenario is illustrated
 557 in Figure 8. The initial condition is a filament of dense
 558 gas which grows by the accretion flow perpendicular to
 559 its axis. Inside the filament, the flow is directed along
 560 its axis. Two over-densities in the filament form two
 561 pairs of similar stars with orbits roughly perpendicular
 562 to the filament, owing to the angular momentum of the
 563 incoming gas. The total masses of both pairs are also
 564 similar because they formed in the same filament and
 565 experienced comparable accretion rates. The pairs ap-
 566 proach each other, driven by mutual attraction and by
 567 the center-of-mass velocities inherited from the parental
 568 gas flow along the filament.

569 Close approach of two pairs and their dynamical
 570 interaction can lead to four different outcomes
 571 (Antognini & Thompson 2016). In the simplest case,

the decay products are just single stars and binaries. If only one star is ejected, a bound triple with three similar components (a triplet) could result. Alternatively, one pair can become very close and merge, leaving a double twin. Finally, a bound 2+2 quadruple can emerge if the dynamical interaction was not too violent or did not happen at all. In all cases the surviving hierarchies bear imprints of chaotic dynamics, namely eccentric orbits with random mutual orientation.

The two massive triplets studied here (ϵ Cha and I 385) match the proposed scenario: their inner orbits have large eccentricities and are not aligned with the outer orbits. HIP 42910, a double twin with eccentric inner orbit, could be a merger product. In contrast, the architecture of the double twin HIP 32475 with aligned quasi-circular orbits better matches the accretion scenario discussed in (Tokovinin 2018a).

A remarkable quadruple system FIN 332 (WDS J18455+0530, HIP 92027, HR 7048, the “tweedles”) illustrates the proposed scenario. It consists of four nearly equal A1V type stars in a 2+2 hierarchy (Tokovinin 2020). Orbits of the two inner twins have periods of 28 and 40 yr and large eccentricities (0.82 and 0.84); moreover, their apsidal axes point in approximately same direction. The outer pair ($P \sim 5$ kyr) moves in the opposite sense and its orbit is definitely misaligned with orbits of the inner pairs. This architecture strongly suggests a past dynamical interaction. If one of the pairs in this system were disrupted and ejected a star, the result could resemble ϵ Cha or I 385.

If ϵ Cha is a product of a decaying 2+2 quadruple, one B-type star should have been ejected. Assuming an ejection speed of 30 km s^{-1} , the star would have traveled a ~ 150 pc distance in 5 Myr. It is almost hopeless to search for the ejected star, it can be located anywhere on the sky. The phenomenon of runaway massive stars is well known, and it is generally accepted that many runaways were ejected from young unstable hierarchies (Hoogerwerf et al. 2000). For effective ejections, other members of these hierarchies must be also massive, and this consideration supports the dynamical scenario of forming massive triples and quadruples.

However, the scenario of triplet formation via decay of a 2+2 quadruple has a serious problem. Ejection of one star with a velocity V causes recoil of the remaining triple with a velocity of $\sim V/3$. The facts that ϵ Cha is close to the neighboring stars in the association and

that I 385 is bound to another star C indicate absence of a fast recoil. Equal masses in triplets can be explained alternatively by accretion from a common gas reservoir while separations between the stars were still large and they moved randomly through the parental core without mutual dynamical interactions; otherwise, one star would have been ejected without a chance to grow further, as discussed by Reipurth (2000). The N-body dynamics may come into play later, when the system have migrated to a more compact configuration and the gas was mostly exhausted; the triple, nevertheless, avoids disruption and continues to move together with its neighbors.

Study of relative motions in hierarchical systems opens a fascinating window on their diversity and suggests formation via several channels, still poorly explored. Extension of such work to a much larger sample of hierarchies is highly desirable. However, long periods and the lack of historic measurements severely restrict potential samples. Indirect statistical approaches using only “instantaneous” data like positions and velocities (e.g. Hwang et al. 2022) are promising for the dynamical study of typical hierarchies with separations of 1–100 au. Long-term speckle monitoring of a large number of resolved hierarchies combined with precise Gaia astrometry will provide input data for these future investigations.

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Facility: SOAR, Gaia

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