

Introduction to Scattering Theory

Application to Telescope Mirrors

Using IRIS 908RS

Part Two

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4. What can we learn from surface scattering theory?

We invoke rapidly some theoretical concepts about surface scattering to make the choices and recommendations understandable by non specialists. Thus, there is no attempt at being exhaustive or even strictly consistent. One should also remember that it applies only to limited cases : pure surface scatterers with defects not much larger than the wavelength.

First level understanding

One can think of a surface scatterer in terms of a superposition of many ruled gratings of various pitch and ruling directions. The first theoretical analysis of scattered light was simply made in term of Fourier analysis of the frequency and amplitude content of the scattering surface. From these early (but conceptually very easy to understand) studies, one should remember the two basic relations :

The grating equation relating the ruling pitch d , the wavelength λ of the light and the incidence (i) and diffraction (θ) directions :

$$\sin(\theta) - \sin(i) = p \frac{\lambda}{d}$$

p is the integer diffraction order. The relation giving the diffracted intensity for small angles between specular and scattering directions ($i \sim \theta$) and for a depth a of the grooves is :

$$I(i) = \left[\frac{2\pi}{\lambda} a \cos(i) \right]^2$$

Of course, one cannot go very far with this treatment. But on top of being very easy to grasp, it already shows characteristic features which will stay with us in the most advanced theories of surface scattering.

- The correct independent angular variable is : $\sin(\theta) - \sin(i)$ which the specialists of scatterometry usually write : $\beta_s - \beta_i$.
- If you change the incidence angle, the scattered intensity drops as the square of the cosine of the angle.

The exact vectorial theory

Considerable amount of work has been devoted to surface scattering. Finally, the most exact treatment is known as the Rayleigh-Rice perturbation theory.

This yields the following result :

$$BSDF(i, \theta) = T \cdot k^4 \cdot \cos^2(i) \cdot \exp\left\{-A \cdot k^2 \cdot [\sin(\theta) - \sin(i)]^2\right\}$$

where T is the transmission of the sample and $k = 2\pi/\lambda$. This equation is somewhat simplified :

- it is written for small angles and projected in the scattering plane
- it does not account for polarization effects

It is remarkable that we find again the two features we had pointed out from the grating theory. A third useful features is comprised in the Rayleigh-Rice relation : the fourth power dependence on wavelength. It can be used to compute the BSDF at a wavelength from the measurements made at another wavelength.

The empirical Harvey relation

As we already pointed out, this is in fact all we need in order to build the most economic and reliable instrument. The Harvey model of surface scatterer follows the law :

$$\frac{BSDF_{\theta,i}}{BSDF_{p,i}} = \left(\frac{|\sin(\theta) - \sin(i)|}{|\sin(p) - \sin(i)|} \right)^m$$

Where p is a conventional “pivot” angle, usually very small, and m (usually negative) is the slope of the log-log diagram of the BSDF as a function of the angular variable

$|\sin(\theta) - \sin(i)|$ or $|\beta_s - \beta_i|$. The Harvey relation is mostly utilized in this logarithmic form because its structure is very simple (a straight line of slope m):

$$Y = Y_0 + m \cdot X$$

where $Y = \log(\text{BSDF})$, $X = \log(|\beta_s - \beta_i|)$ and Y_0 comprises the constant terms.

One notice that if two BSDF measurements are performed on a sample at two different angles, the Harvey relation allows to compute $b = \text{BSDF}_{p,i}$ and the slope m . The choice of p is arbitrary and can be chosen as a significant angle for the particular application one has in mind. For IRIS 908RS, the incidence angle is $i = 45^\circ$. DMO has chosen $p = 45^\circ - \epsilon_0$, $\epsilon_0 = 1.75^\circ$, and the instrument will compute b and m from the measurements at $\theta = 0^\circ$ (45° from the specular direction) and at $\theta = 30^\circ$ (15° from the specular direction). These data then allow to compute the BSDF at any angle.

The Harvey relation turns out to be an excellent representation of the scattering behavior of well behaved surface scatterers such as clean polished mirrors with small defects. If the scatterer is not well behaved (as a dusty mirror for instance), the Harvey law still represents good portions of the angular dependency of the BSDF, but is less accurate for angles outside the range of the two calibration angles.