Introduction to Scattering Theory

Application to Telescope Mirrors

Using IRIS 908RS

Part Three

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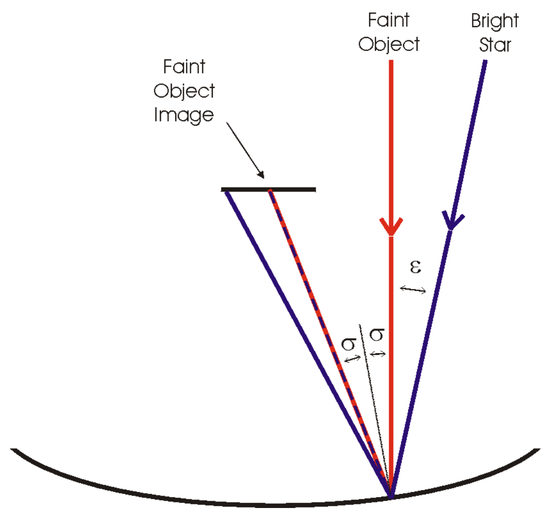
5. Application of the Harvey law to telescope mirrors



A typical astronomy application would be the following. Assume that you measure a faint stellar-like object close to a bright star with a pixel detector (see picture here above). Because the mirror is not perfect nor clean, a part of the light from the bright star will be scattered in the direction of the faint object image. It is a parasitic source of light. Thanks to the BRDF function of the mirror, it is possible to estimate what is the magnitude of the bright star scattered light and compare it to the magnitude of the faint objet.

Let say that the pixel size covers a field of view of a solid angle α x α (α is an angle in radians. To convert second of arc to radians use the following formula : radians = second of arc x π / 648000). The distance between the bright star and the faint object is ε. From the measurement performed with the IRIS 908RS, the Harvey coefficients b and m are known.

On the picture here above, we consider what is happening at a certain location on the mirror. At that location, the slope of the mirror is σ. We will see that this slope is without importance in the calculation. What we want to know is how much light coming from the bright star is scattered in the faint object image direction. So the incident angle of the bright star is $\sigma + \varepsilon$. The scattered light angle is σ. Now we can calculate the BRDF function for those incident and scattering angles :

$$BRDF = b\left(\frac{\left|\sin(\sigma+\varepsilon)-\sin(\sigma)\right|}{\left|\sin(\sigma+\varepsilon_0)-\sin(\sigma)\right|}\right)^m$$

where ε_0 is the pivot angle of the Harvey law (ε_0 = 1,75° for IRIS 908RS). Because all angles are very small, the sinus of the angles are the angles themselves. The BRDF formula can be simplified :

$$BRDF = b\left(\frac{\varepsilon}{\varepsilon_0}\right)^m$$

As previously mentioned, the slope of the mirror has disappeared from the formula. The only important factor is the distance between the faint object and the bright star (ε). Because the BRDF is defines as follow :

$$BRDF = \frac{\delta P_s}{P_i \delta\Omega_s \cos(\sigma)}$$

it is possible to calculate how much of the bright star light is polluting the faint object viewing. In this formula, $\cos(\sigma)$ is nearly 1 as the angle σ is very small. $\delta\Omega_s$ is the solid angle of scattered light per pixel. In our case it is the pixel size α x α. P_i is the power of the bright star. Combining BRDF definition and Harvey law, one sees that :

$$\frac{\delta P_s}{P_i} = \alpha^2 b\left(\frac{\varepsilon}{\varepsilon_0}\right)^m$$

The same formula expressed in magnitude is :

$$\text{Scattered Light Magnitude} = \text{Bright Star Magnitude} - 2.5\log\left(\alpha^2 b\left(\frac{\varepsilon}{\varepsilon_0}\right)^m\right)$$

Example

The pixel size is 0.5" x 0.5". The bright object is of magnitude 2; what amount of (parasitic) light from the bright object is polluting your faint object if the distance between the two object is 5" and the IRIS scattering measurement on the primary yields the values m = -2.168 and b = 0.3618 ?

In the formula here above, $\alpha = 0.5 \times \pi / 648000 = 2.42\ 10^{-6}$ radian and $\varepsilon = 5 / 3600 = 1.39\ 10^{-3}$ °. ε_0 is a constant value (1,75° for IRIS 908RS). Therefore

$$\text{Scattered Light Magnitude} = 2 - 2.5\log\left((2.42\,10^{-6})^2\,0{,}3618\left(\frac{1.39\,10^{-3}}{1.75}\right)^{-2.168}\right) = 14.4$$

The light scattered by the bright star has a magnitude of 14.4 per pixel.

Limitations of the Harvey law

The main drawback of the Harvey representation is that it breaks down at very small angles since its mathematical formulation exhibits a singularity at $\theta = i$. This makes the result of the preceding section approximate, since the BRDF at 5" from the specular direction has been deduced from measurements made at 15° and 45° from the specular direction. The result should not be used in correcting the photometry of the faint object, but just to make a preliminary estimate of what can be expected. For instance, such computation will show immediately whether the mirror cleanliness is critical for a given observation. The actual photometric calibration of the scattering should be performed by measuring an empty field at the same distance of the bright star.