Optical Solutions DMO

Introduction to Scattering Theory

Application to Telescope Mirrors

Using IRIS 908RS

Part One

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1. What causes optical scattering in a telescope?

A perfect optical surface deflects an optical beam specularly, i.e. in straight lines following Snell's laws. A parallel beam of diameter D should ideally be focused at one point by a perfect telescope mirror.

Because of **diffraction** phenomenon at the entrance of the telescope, the power of the inlet beam is spread onto a small area instead of a single point. The diffracted angular deflection with respect to purely specular directions are very small; they depend on the beam diameter D (it is the telescope diameter) and the wavelength λ so most of the power falls inside a solid angle circle of an angular radius $1.22*\lambda/D$. This is known as the Airy disk for a circular opening. Since this is the minimum image dimension for perfect conditions, it is used as a natural scale in our discussion.

If the surface is not perfect and the **defects are smaller than a few wavelengths** (and smaller than the coherence length of the source), some additional power will fall outside the Airy disk. This type of scattering yields to mathematical analysis, allowing to compute the relation between the physical shape of the defects (micro-roughness) and the resulting optical scattering. This is typically the case of a good optical surface such as a clean optical mirror.

If the **defects are very large compared to the wavelength**, power is scattered at large angles with respect to the specular direction. There is no theory for accounting for this type of scattering, but only empirical relations (between physical defects size and distribution, and scattering). One can visualize the surface as covered by thousands of microscopic mirrors of all shapes and various dimensions placed at all angles with the mean surface. There cannot be a theory because there are as many scattering functions as there are distributions of micro-mirrors. This is the case of dusty optical mirrors or of ground glass. Hence, it is improper to convert the scattering measurements of a dusty mirror into micro-roughness values.

2. How do we measure the scattering properties of a surface?

The scattering properties of a surface is measured by a scatterometer. This instrument essentially throws a well collimated beam on the test surface at a well defined incidence angle, and one or several detectors detect and measure the scattered light. These detectors are mounted on arms so that the scattering angle can be varied. The scattering solid angle is defined by baffles and stops.

Usually it is very difficult and inaccurate to measure directly the scattering at angles smaller than 2 or 3°. This is due to straylight within the instrument. The latter is mainly produced by dust and stains on the front collimating lens and, hence is essentially variable. If it is not too important, and if we measure the transmission scattering, the instrument stray light can be approximately removed by calibration, measuring without sample. This calibration procedure, of course does not work for reflective measurements since then, no measurement is possible without sample. Also, portable instruments are bound to utilize short path beams with relatively large apertures; hence, baffling for small angles is impossible.

In reflective measurements of mirrors before aluminizing, the measurement is usually polluted by a certain amount of volume scattering by the bulk of the blank. This difficulty disappears of course when the mirror is aluminized.

3. What is the usual way of expressing the scattering of an optical surface ?

There are basically two related ways. First the angle-resolved scattering known as Bi-directional Scattering Distribution Function (**BSDF)**. This function is the relation between the light power received within a solid angle in a certain scattering direction depending on the beam incidence angle and its power. The second is the Total Integrated Scattering (**TIS)**. It is a global scattering function that says how much of the incident power is scattered in all direction depending on the incidence to the surface. The TIS function cannot be used to quantify the effect of scattering on the image sharpness in a telescope.

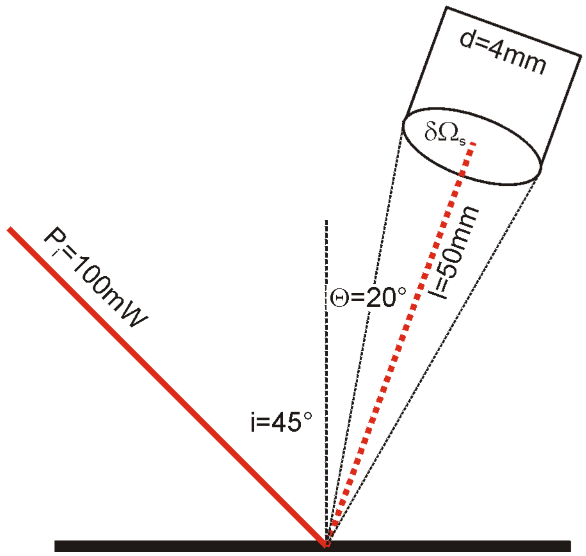
The BSDF is mostly utilized in its simplified form where only the incidence plane is considered :

$$BSDF(i,\vartheta) = \frac{\delta P_s(i,\vartheta)}{P_i \delta\Omega_s \cos(\vartheta)}$$

In this expression, i is the incidence angle of the source beam (from the normal to the scattering surface), θ is the scattering direction (from the normal), P_i is the power in the incident beam, δP_s is the power in the scattered beam inside the solid angle $\delta\Omega_s$ around the direction θ. It is assumed that the whole illuminated surface is seen by the scattering detector or that the powers are expressed per unit area of the scatterer. The BSDF is expressed in $steradians^{-1}$. These measurements are well known and documented in optics *[Jean M. Bennet and Lars Mattsson, Surface roughness and scattering, OSA, Washington D.C.] [John C. Stover, Optical Scattering, SPIE Opt. Eng. Press]*.

Let us consider an example. The incident beam on a surface is 100 mW. Its angle from the normal direction is 45°. There is a sensor located at 50 mm from the surface at an angle of 20° from the normal. The sensor is circular and its diameter is 4 mm.

How much power will reach the sensor if the BSDF(45°,-20°) of the surface is 0.003% of steradians^{-1} ?



The surface of the sensor is $\pi d^2/4$. The solid angle ($\delta\Omega_s$) of the sensor related to the incident spot on the surface is 4π(surface of the sensor)/(surface of the sphere) = $\pi d^2/4l^2$ = 0.005027 steradians. From the BSDF definition, the power collected by the sensor is $\delta P_s(45°,20°)$ = BSDF(45°,20°) x Pi x $\delta\Omega_s$ x cos(20°) = 0.00003 x 100 x 0.005027 x 0.939692 = 0,0000142 mW. This example shows that if the BSDF is known, it is possible to calculate how much light is going in any direction for a given incident beam power. The IRIS 908RS from DMO calculates the BSDF. Later on, it will be shown how to apply this for telescope mirrors.

The BSDF is a monotonous decreasing function which covers several orders of magnitudes when going from small angles with respect to the specular direction (the direction where the beams proceeds when there is no scattering) to large angles. When the scattering is measured on the side of the transmitted beam, one calls the function the BTDF, and when measured on the side of the reflected beam, it is called the **BRDF** (so that the BSDF of a transparent surface is the junction of its BTDF and its BRDF).

Relation between BSDF and radiance

One can rearrange the BSDF definition equation to show its relation to the radiometric quantity "radiance" by replacing the total incoming power by the power per surface unit δa:

$$BSDF(i,\vartheta)\cdot\frac{\delta P_i}{\delta a}\cdot\delta\Omega\cdot\delta a\cdot\cos(\theta)=\delta^2 P_s(i,\theta)$$

one see that the radiance is :

$$L_e(i,\theta)=BSDF(i,\theta)\cdot\frac{\delta P_i}{\delta a}$$